Efficiency in the Mortgage Market: The Borrower's Perspective

John M. Quigley* and Robert Van Order**

This paper uses mortgage history data from the Federal Home Loan Mortgage Corporation to analyze the prepayment behavior of homeowners and to test whether borrowers exercise their prepayment options in a manner consistent with contingent claims models. A variety of hazard models are estimated from individual data on more than 6000 mortgages issued during the 1976–1980 period. In these models, it is clear that the extent to which the prepayment option is "in the money" has a strong effect on behavior. However, it is less clear that the option is exercised quite as ruthlessly as the theory predicts.

INTRODUCTION

It is by now widely accepted that a fruitful way of analyzing and pricing mortgages is to view them as ordinary debt instruments with various options attached to them. Default is a put option; the defaulter sells his house back to the lender in exchange for eliminating the mortgage obligation. Prepayment is a call option; the borrower exchanges the unpaid balance on the debt instrument for a release from further obligation. Analogously, caps and floors on adjustable-rate mortgages can be formulated as options. Dunn and McConnell [5], Buser and Hendershott [2], Kau et al. [10], and Brennan and Schwartz [3], are among the many papers that apply recent contingent claims...
models to pricing mortgages. Much of this is based on the work of Black and Scholes [1] and Cox, Ingersol and Ross [4]. Hendershott and Van Order [8] survey some of these results. Some of the approaches used by Wall Street firms to model prepayment and to price mortgages are discussed in the June 1988 special issue of the Journal of Real Estate Finance and Economics.

This paper uses mortgage history data from the Federal Home Loan Mortgage Corporation (Freddie Mac) to estimate a prepayment function and to test whether borrowers exercise their prepayment options in a manner consistent with the optimal strategy developed with contingent claims models. The key variable in the model is the difference between the current market value and the par value of the mortgage, i.e., the extent to which the option is "in the money." This variable, which reflects principally the difference between the coupon rate on the mortgage and current market rate, turns out to have a very strong effect on prepayments, as predicted. However, it is less clear that the option is exercised quite as ruthlessly as the theory predicts.

OPTIMAL PREPAYMENTS OF MORTGAGES

Well-informed borrowers in a perfectly competitive market will always prepay mortgages when they can increase their wealth by engaging in such behavior. Absent transactions costs, borrowers in perfect markets can always increase their wealth by prepaying when the market value of the mortgage exceeds the outstanding balance on the loan. Note that the market value of the mortgage exceeds the present value of the remaining payment stream because the present value of the remaining payments ignores the option to prepay at some subsequent date. Consequently, even if interest rates fall so that the present value of the remaining payments is greater than the outstanding balance (i.e., the option is "in the money"), it may not be optimal to exercise the option. In so exercising it, the borrower would lose the option to exercise it later.

To solve the problem of when to exercise the option, the contingent claims model begins by specifying the underlying state variables that determine the price of a security. For a default-free, callable mortgage, these are interest rates. The value of a mortgage is given by \( M(r,a,T) \), where \( r \) is a vector of interest rates, \( a \) is the age of the mortgage, and \( T \) is the time at
which it matures. A standard arbitrage argument is sufficient to derive the equilibrium conditions for determining the function $M$ (see Brennan and Schwartz [3] and Hendershott and Van Order [8]). This condition is a second order partial differential equation which specifies that the expected return on the security (that is, the coupon return plus capital gains) must equal the risk-free rate of return plus a risk adjustment. The condition applies to any claim that is contingent on the underlying state variables.

An infinite number of functions satisfy the partial differential equation (depending on boundary conditions), which reflects the infinite number of ways that coupon plus capital gain can equal the required expected return. By incorporating the optimal call strategy, the function appropriate for a callable bond can be determined.

Suppose that just one interest rate, the short rate $r$, matters.' In this case, the optimal call strategy, given $a$, is a value of $r$, $r^*(a)$, at which the mortgage is called. For $r^*$ to be optimal, it must be chosen to minimize the value of the mortgage (which maximizes the borrower's net worth), subject to the condition that the value of $M$ equal the remaining balance when the call is exercised.

Figure 1 indicates the optimal call strategy for a mortgage of given age $a$. The upper dotted curve represents the inverse relationship between price and interest rates for a noncallable mortgage, while the horizontal line ($Par$) represents the par value of the mortgage. For a callable mortgage, the optimal exercise strategy is represented by the line marked $Z$, the lowest curve that satisfies the equilibrium condition and is tangent to the horizontal line indicating par. The tangency incorporates the optimizing strategy because, for an interior solution, the curve that minimizes value must be tangent to the par line. This determines $r^*$, the critical rate. The curve thus gives the market relationship between price and interest rates and indicates how the market value anticipates the call before it is exercised. (See Hendershott and Van Order [8] for a more extensive discussion.)

Dunn and McConnell [5], Quigley [11], and others have analyzed homeowner mobility and nonfinancial motives for

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'it is much more difficult to calculate solutions if there is more than one state variable. Most papers [e.g., Dunn and McConnell [5], Buser and Hendershott [2] and Cox, Ingersoll and Ross [4]] assume that only one state variable, the short rate, matters, and that this rate determines the entire term structure of interest rates. Brennan and Schwartz [3] use two state variables, a long rate and a short rate.
prepayment. Most conventional mortgages are not assumable, forcing homeowners to prepay low rate loans if they move.\footnote{As a result of changes in the law and also in Freddie Mac policy, the loans in our sample were assumable during some of the period of observation (particularly in the early years) and nonassumable in other years. Because the rules for assumability are complicated and vary by state, we have not adjusted for them. See Quigley \cite{11} or Preiss \cite{12} for a discussion.} Even in a frictionless market, this introduces other demographic and economic factors into the analysis, implying an underlying level of prepayments, even with constant interest rates. Note, however, that in the absence of transactions costs borrowers with nonassumable mortgages will still refinance in a ruthless way if interest rates fall. There will still exist a critical value of \( r \) that triggers immediate prepayment, but there will be a positive probability of prepayment even if rates rise.

Transactions costs can be introduced as a wedge between the borrower's payment and the lender's receipt. The borrower buys back the mortgage at \( par + c \), where \( c \) is the transaction cost. The decision calculus is unchanged, but the tangency is now with the
par + c line, which determines a new critical \( r, r(1)^* \), and the optimal strategy is represented by the curve marked \( X \).

The lender's curve, which represents the price of the security in the market, lies below the borrower's curve because the lender only receives par at exercise. Hence, at exercise (at \( r[1]^* \)) he gets par while the mortgage is worth more than par to the borrower. Thus the relationship between market price and interest rates (the curve indicated by \( Y \)) now has an upward sloping part, just before exercise.

Even with transaction costs, the model still implies rather ruthless exercise. Everyone should prepay at about the same \( r^* \), unless transaction costs vary across individuals.

The discussion so far has ignored the possibility of default. We do not bring default formally into the choice model estimated below, but we should note that loans that are about to default, i.e., those with negative home equity, will not refinance, so that a more general model of prepayments would require both a critical \( r \) and a critical house price. (Kau et al. [10] develop a model where both default and prepayment options can be exercised.) Because default is relatively rare (in our sample, default rates were under 0.5% per year while prepayment rates were typically ten to fifteen times as large), we ignore it in our prepayment model. We do however include the initial equity in the property (a proxy for the probability that equity will be negative in the future) as an explanatory variable in our statistical work.

We do not know the value of \( r^* \) without solving the entire model. We do know that, without transactions costs, \( M = \text{par} \) at \( r^* \), but we also do not know \( M \) until we solve the entire model. However, \( M \) is equal to the value of a noncallable mortgage, \( \bar{M} \), which can be computed as the present value of the remaining payments, plus the value of the call option, which we do not know. Consider the ratio \( A = (\bar{M} - \text{Par})/\text{Par} \). It represents the extent to which the option is in the money. The ratio will be larger the greater is the difference between the current interest rate and the rate at which the instrument is written; the ratio will also be larger the greater is the term to maturity. This ratio represents the percent savings (neglecting any transactions costs) that could be achieved by refinancing of the outstanding balance, ignoring the loss from foregoing the option to prepay later.

The "ruthless" prepayment model predicts that there is some critical value for \( A, A^* \) corresponding to \( r^* \), at which the prepayment function becomes infinite. This is presented in Figure 2 where the function is vertical at \( A^* \). Without trans-
actions costs, the extent to which \( A^* \) exceeds zero depends upon
the extent to which the option must be in the money before it is
optimal to exercise it (and to forego the option to exercise it
later). The addition of transactions costs will raise \( A^* \) further.
The transactions costs of prepaying are on the order of 3% of the
mortgage balance. The extent to which \( A^* \) exceeds this level will
vary with the value of the prepayment option, and more
generally with those household characteristics that affect mobi-
licity prospects and the likely term (see Quigley [11]).

Figure 3 depicts the cumulative prepayment function after
some shock at age \( a^* \). The function shifts quickly, with cumu-
ulative prepayments rising to one at age \( a^* \).

THE MODEL

We postulate a hazard relationship of the form

\[
H(a) = \rho(a) = \lambda(a) \exp \{ \Sigma \beta \}
\]

where the underlying hazard \( H(a) \), as a function of the age of the
mortgage, is merely the conditional probability \( \rho \) of prepayment.
at age $a$. The conditional probability is related to $\lambda(a)$, a normal or "baseline" hazard (that represents the prepayment behavior at constant interest rates for holders of nonassumable mortgages), and $x$, a vector of explanatory variables including the initial equity and the variable of interest, $A$. The initial equity is measured by two dummy variables representing loan-to-value (LTV) ratios between 80% and 90%, and above 90%, respectively. The variable denoted by $A$ depends upon the age and term of the mortgage, the interest rate at which it is written and the current mortgage interest rate. The critical value $A^*$ is, of course, unobserved.

Our fitted model is thus of the form

$$
\rho = \lambda(a) \exp \{\beta_1 (A-A^*) + \beta_2 LTV_2 + \beta_3 LTV_3\} \\
= [\lambda(a) \exp \{-\beta_1 A^*\}] \exp \{\beta_1 A + \beta_2 LTV_2 + \beta_3 LTV_3\} \\
= L(a) \exp \{\beta_1 A + \beta_2 LTV_2 + \beta_3 LTV_3\}.
$$

(2)

As indicated by the square brackets, we cannot identify $\lambda(a)$ separately from $\exp(-\beta_1 A^*)$. Thus for positive values of $\beta_1$, the
baseline hazard implied by our model is less than \( \lambda(a) \). The method of partial likelihood (see Kalbfleisch and Prentice [9]) makes it possible to obtain consistent estimates of the parameters, \( \beta_1, \beta_2 \) and \( \beta_3 \), of the model without knowledge of \( L(a) \).

The proportionality of the hazard relationship can, of course, be tested by estimating

\[
\rho = L'(a) \exp \{ \beta_1 A + \beta_2 LTV_2 + \beta_3 LTV_3 \}
\]

\[
= L'(a) \exp \{ \beta_1 A + \beta_1' A \log a + \beta_2 LTV_2 + \beta_3 LTV_3 \},
\]

and testing whether \( \beta_1' \) is different from zero.

RESULTS

During the 1976-80 period, mortgages were issued with coupons varying between 8% and 14%. Since 1983, new issue coupons have again varied within this range. Hence, mortgages originated during the sample period have had the prepayment option well in the money and well out of the money.

Table 1 presents some illustrations of the value of \( A \) observable during this period. For instance, as mortgage interest rates fluctuate in the 8% to 13% range, the savings from prepayment can be as high as 51% for newly issued thirty-year mortgages and about 12% for mortgages with only five years remaining to maturity. For a given coupon rate and term to maturity, the savings from prepayment as a fraction of the mortgage balance is roughly linear in current interest rates.

Table 2 presents summary data on the samples and the results of the estimations. The top panel of Table 2 indicates the data available for each of the five Freddie Mac regions. These data represent a 2% random sample of thirty-year, fixed-rate mortgages originated during the 1976-1980 period and purchased by Freddie Mac. More than half of the almost 6,400 observations come from the Western region, reflecting the geographical distribution of Freddie Mac's business during the period. Some 48% of the loans had prepaid by March 1989, implying an average annual prepayment rate of 6% or 7%.

The lower panel of the table presents estimates of proportional hazards models relating the time-varying parameter \( A \) to the conditional probability of prepayment. These "Cox-Regression" (see Kalbfleisch and Prentice, [9]) were estimated by partial likelihood methods. The models are estimated separately for the
### TABLE 1

**Savings from Prepayment (A) as Fraction of Remaining Mortgage Balance**

<table>
<thead>
<tr>
<th>Current Rate</th>
<th>8.0%</th>
<th>9.0%</th>
<th>10.0%</th>
<th>11.0%</th>
<th>12.0%</th>
<th>13.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Remaining term: 360 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>9.0%</td>
<td>−0.09</td>
<td>0.00</td>
<td>0.09</td>
<td>0.18</td>
<td>0.28</td>
<td>0.37</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.16</td>
<td>−0.08</td>
<td>0.00</td>
<td>0.09</td>
<td>0.17</td>
<td>0.26</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.23</td>
<td>−0.16</td>
<td>−0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>12.0%</td>
<td>−0.29</td>
<td>−0.22</td>
<td>−0.15</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td>13.0%</td>
<td>−0.34</td>
<td>−0.27</td>
<td>−0.21</td>
<td>−0.14</td>
<td>−0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>B. Remaining term: 300 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.09</td>
<td>0.18</td>
<td>0.27</td>
<td>0.36</td>
<td>0.46</td>
</tr>
<tr>
<td>9.0%</td>
<td>−0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.17</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.15</td>
<td>−0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.16</td>
<td>0.24</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.21</td>
<td>−0.14</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.07</td>
<td>0.15</td>
</tr>
<tr>
<td>12.0%</td>
<td>−0.27</td>
<td>−0.20</td>
<td>−0.14</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>13.0%</td>
<td>−0.32</td>
<td>−0.26</td>
<td>−0.19</td>
<td>−0.13</td>
<td>−0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>C. Remaining term: 240 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.08</td>
<td>0.15</td>
<td>0.23</td>
<td>0.32</td>
<td>0.40</td>
</tr>
<tr>
<td>9.0%</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.07</td>
<td>0.15</td>
<td>0.22</td>
<td>0.30</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.13</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.19</td>
<td>−0.13</td>
<td>−0.07</td>
<td>0.00</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>12.0%</td>
<td>−0.24</td>
<td>−0.18</td>
<td>−0.12</td>
<td>−0.06</td>
<td>0.00</td>
<td>0.06</td>
</tr>
<tr>
<td>13.0%</td>
<td>−0.29</td>
<td>−0.23</td>
<td>−0.18</td>
<td>−0.12</td>
<td>−0.06</td>
<td>0.00</td>
</tr>
<tr>
<td>D. Remaining term: 180 months</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.19</td>
<td>0.26</td>
<td>0.32</td>
</tr>
<tr>
<td>9.0%</td>
<td>−0.06</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.11</td>
<td>−0.06</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.16</td>
<td>−0.11</td>
<td>−0.05</td>
<td>0.00</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>12.0%</td>
<td>−0.20</td>
<td>−0.15</td>
<td>−0.10</td>
<td>−0.05</td>
<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>13.0%</td>
<td>−0.24</td>
<td>−0.20</td>
<td>−0.15</td>
<td>−0.10</td>
<td>−0.05</td>
<td>0.00</td>
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<tr>
<td>E. Remaining term: 120 months</td>
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<td></td>
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<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.14</td>
<td>0.18</td>
<td>0.23</td>
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<td>9.0%</td>
<td>−0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.08</td>
<td>−0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.12</td>
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<td>−0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.08</td>
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<td>12.0%</td>
<td>−0.15</td>
<td>−0.12</td>
<td>−0.08</td>
<td>−0.04</td>
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<td>0.04</td>
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<tr>
<td>13.0%</td>
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<td>−0.15</td>
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<tr>
<td>F. Remaining term: 60 months</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>9.0%</td>
<td>−0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>10.0%</td>
<td>−0.05</td>
<td>−0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td>11.0%</td>
<td>−0.07</td>
<td>−0.05</td>
<td>−0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
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<tr>
<td>12.0%</td>
<td>−0.09</td>
<td>−0.07</td>
<td>−0.04</td>
<td>−0.02</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>13.0%</td>
<td>−0.11</td>
<td>−0.09</td>
<td>−0.07</td>
<td>−0.04</td>
<td>−0.02</td>
<td>0.00</td>
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</tbody>
</table>
TABLE 2
Hazard Models of Mortgage Prepayment by Region
(asymptotic t-ratios in parentheses)

<table>
<thead>
<tr>
<th>Summary Data</th>
<th>West</th>
<th>Southwest</th>
<th>Southeast</th>
<th>Northeast</th>
<th>North Central</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Observations</td>
<td>3,531</td>
<td>1,052</td>
<td>273</td>
<td>799</td>
<td>720</td>
<td>6,375</td>
</tr>
<tr>
<td>Number of Prepayments</td>
<td>1,762</td>
<td>386</td>
<td>145</td>
<td>433</td>
<td>336</td>
<td>3,062</td>
</tr>
</tbody>
</table>

Parameter Estimates
Model 1
\[ A \]
\[
\begin{array}{lrrrrrrr}
   (21.76) & (5.37) & (3.85) & (7.11) & (27.01) & & & \\
\end{array}
\]
\[
\chi^2 = 494.25, 28.94, 15.10, 137.64, 51.33, 791.23
\]

Model 2
\[ 80\% \leq LTV \leq 90\% \]
\[
\begin{array}{lrrrrrr}
   (1 = yes) & 0.017 & -0.003 & 0.040 & 0.044 & -0.052 & 0.003 \\
   (2.23) & (0.03) & (0.19) & (0.37) & (0.44) & (0.60) & & \\
\end{array}
\]
\[
\begin{array}{lrrrrrr}
   90\% \leq LTV & -0.372 & -0.195 & 0.004 & -0.073 & -0.295 & -0.201 \\
   (1 = yes) & (2.69) & (1.55) & (0.02) & (1.77) & (3.65) & & \\
\end{array}
\]
\[
\begin{array}{lrrrrrrr}
   (21.77) & (5.40) & (3.84) & (11.05) & (7.05) & & (27.55) \\
\end{array}
\]
\[
\chi^2 = 502.52, 31.89, 15.13, 138.64, 54.81, 806.59
\]

Model 3
\[ 80\% \leq LTV \leq 90\% \]
\[
\begin{array}{lrrrrrr}
   (1 = yes) & 0.016 & -0.004 & 0.042 & 0.049 & -0.042 & 0.009 \\
   (0.31) & (0.03) & (0.30) & (0.42) & (0.35) & (0.20) & \\
\end{array}
\]
\[
\begin{array}{lrrrrrr}
   90\% \leq LTV & -0.302 & -0.192 & -0.003 & -0.064 & -0.276 & -0.193 \\
   (1 = yes) & (2.60) & (1.51) & (0.02) & (0.53) & (1.65) & (3.49) \\
\end{array}
\]
\[
\begin{array}{lrrrrrrr}
   A & 90.730 & 89.600 & 191.453 & 107.522 & 116.075 & 101.563 \\
   (7.93) & (3.39) & (4.04) & (5.24) & (3.19) & & (12.05) \\
\end{array}
\]
\[
\begin{array}{lrrrrrr}
   A \log \alpha & -31.426 & -30.077 & -91.163 & -41.644 & -47.509 & -37.897 \\
   (5.48) & (2.70) & (4.04) & (3.99) & (3.30) & & (8.89) \\
\end{array}
\]
\[
\chi^2 = 540.59, 41.08, 35.52, 154.52, 69.77, 907.26
\]

five regions. The table also presents results for the pooled sample of mortgages across regions.

Model 1 includes only the variable measuring the extent to which the option is in the money; Model 2 includes dummy variables (fixed covariates) reflecting the initial equity in the house as well as the time varying parameter A. Model 3 allows for non-proportionality in the hazard.

As the results for Model 1 indicate, the extent to which the option is in the money exerts a powerful effect upon the prepayment hazard. Model 2 provides weak but consistent
evidence that the initial equity affects prepayment choices. The variable signifying $LTVs$ in excess of 90% is negative in five of the six replications. The coefficient is statistically significant for the western region and for the pooled national model. A likelihood ratio test comparing Models 1 and 2 indicates that the measures of initial equity are jointly significant in four of the six stratifications.

The coefficient of $A$, the key variable of interest, is highly significant and quite large in magnitude. Its impact upon prepayment probabilities, of course, depends upon the unobserved value of $A^*$. For reasonable values of $A^*$, the extent to which the option is in the money has a substantial impact on prepayment. For example, if transactions costs are 3% and if the value of the option is 5% so that $A^*$ is 0.08, then a value of $A$ of 0.10 increases the prepayment hazard from its baseline by a factor of $\exp(27.331 \times (0.10 - 0.08))$ or by 73%.

Model 3 suggests that the relationship between $A$ and the prepayment hazard is not proportional. For less seasoned mortgages, the probability of prepayment is substantially higher than for those with shorter terms to maturity, even when identical percentage savings are possible from prepayment. For example, if again transactions costs are 3% and the value of the option is 5%, then a value of $A$ of 0.10 increases the prepayment hazard for two-year-old mortgages by 232% (using the estimates for the US as a whole in the last column of Model 3). For ten-year-old mortgages the results indicate that the prepayment hazard is increased by 33% from its baseline.

Although the parameters $\beta^*$ and $\beta_1$ are rather precisely estimated, given the $t$-ratios reported for Model 3, they probably overestimate the non-proportionality of the prepayment response over the entire term of the typical thirty-year mortgage. According to the estimates, the proportionate shift is actually negative after the fourteenth year. However, no mortgage in our sample is older than thirteen years. The sharp non-proportionality does, however, “fit” this body of truncated data.

**IMPLICATIONS**

As indicated in equation 1, estimates of the proportional shifts in the prepayment hazard are expressed relative to some baseline hazard, $L(a)$, which may vary with the age of the mortgage. We
made several attempts to parameterize and estimate the baseline hazard. These attempts yielded inconclusive results.\(^3\)

At least in financial institutions, however, there exist widely used rules of thumb about the magnitude of the baseline hazards. For example, baseline prepayment assumptions are described and discussed in analyses of collateralized mortgage obligations in terms of "PSA survival" rates. Figure 4 presents the cumulative prepayments anticipated under these "PSA survival" rates.\(^4\)

Relative to this baseline hazard, Figure 4 presents the prepayment patterns predicted by Models 1 and 3 (using the pooled model for the US as a whole) for \(A\) values of 0.12, and 0.16 respectively, for an assumed value of \(A^*\) equal to 0.08. In constructing the figure, we assume that five years after the mortgages are written, interest rates decline so that the borrower can gain \(A\) percent by prepaying. As the figure illustrates, the level of prepayment for these new mortgages is quite sensitive to the value of \(A\). For a value of \(A\) equal to 16\% (roughly, a drop in interest rates of about two percentage points on a mortgage with twenty-five years to maturity), Model 1 predicts that almost 65\% of mortgages would have prepaid in year six. In contrast, under the PSA standard about 22\% of mortgages would have prepaid by the end of year six.

For Model 3, the change in prepayments arising from the value of \(A\) is even larger. The model predicts that essentially all the mortgages prepay by the end of year seven.

\(^3\)For example, we assumed the baseline was log-logistic:

\[
L(\alpha) = \alpha \alpha^+, \n\]

and estimated an extended form of Model 1 by maximum likelihood techniques:

\[
\rho(\alpha) = \alpha \alpha^{-1} \exp [x, A].
\]

For several of the regions, the estimates did not converge. For others, the exponent was estimated to be zero. For example, for the western region where the sample is largest, the results for Model 1 are:

\[
\rho = 0.015 \exp [28.524 A], \chi^2 = 494.25.
\]

where the t ratio of the log-logistic parameter 0.136 is 0.001. These results, which imply a constant baseline hazard, are consistent with the "PSA survival" rule of thumb.

\(^4\)The underlying hazard assumed under the "PSA survival" is a linear increase from zero to 6\% for two years and a constant 6\% hazard rate thereafter.
Figure 4

Prepayment Patterns for A Values of 16% and 12% after Five Years
(models 1 and 3)

Cumulative Prepayments (A=16%)

Cumulative Prepayments (A=12%)
Figure 5
Prepayment Patterns for $A$ Values of 16% and 12% after Nine Years
(Model 1 and 3)

Cumulative Prepayments ($A=16\%$)

Cumulative Prepayments ($A=12\%$)
For a smaller value of $A$, the effect on prepayments is less pronounced. As indicated in the lower panel of the figure, a value of $A$ of 12% in year five leads to an increase in the cumulative prepayment rate from 28% to almost 50% after seven years, using either Model 1 or Model 3.

Figure 5 presents simulations of the same changes in the value of $A$, from zero to 16% and 12% respectively, after nine years. The responses, in terms of prepayments, are estimated to be much smaller, but are still substantial.

Notwithstanding the continuous prepayment response to changes in interest rates, the results do suggest that there is a significant acceleration in prepayment (for five-year-old mortgages) as market interest rates decline by 150 to 200 basis points below mortgage coupon rates. This is roughly consistent with rules of thumb suggesting that it is not worthwhile to refinance unless market interest rates decline by 200 basis points. The results also suggest that the option has to be in the money by about 10% before it is exercised rapidly.

CONCLUSIONS

The results reported in this paper document that borrowers behave qualitatively as would be predicted by the options approach to prepayment. It is less clear whether borrowers behave in a "ruthless" fashion, but the finding that it takes a decline of 200 basic points to accelerate rapidly prepayments suggests that the prepayment option is not exercised ruthlessly.

However, our data do not permit a more thorough investigation because the big movement in interest rates during the sample period was upward. The mortgages in the sample had coupon rates of 8% to 12%; rates went up to 16% to 17% in the early 1980s and did not decline as low as 10% until 1986. Hence, our sample does not permit a detailed investigation of prepayment "burnout" over several cycles.\(^{5}\)

That we observe behavioral responses to interest-rate increases is also clear from the parameter estimates. Apparently

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\(^{5}\)The "burnout" phenomenon reported by many market analysts holds that: when rates fall there is a rapid increase in prepayments by the more sophisticated "rational" borrowers; so that when rates fall a second time prepayments by the less sophisticated borrowers remaining in any pool are less responsive.
many borrowers postponed moving or trading up when interest rates increased, and they exercised the prepayment option when interest rates declined subsequently.

A previous version of this paper was presented at the AEA/AREUEA joint session on efficiency in real estate markets, Atlanta, Georgia, December 1989. This paper could not have been completed without the assistance of Carl Mason and Bill Shauman. We are grateful for the comments of Ralph Braid. Quigley’s research was supported by the Center for Real Estate and Urban Economics, University of California, Berkeley.

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