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By

Jiro Yoshida

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Essays on Equilibrium Asset Pricing and Investments

by

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Essays on Equilibrium Asset Pricing and Investments

Abstract

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Asset prices have tremendous impacts on economic decision-making. While substantial progress has been made in research on financial asset prices, we have a quite limited understanding of the equilibrium prices of broader asset classes. This dissertation contributes to the understanding of properties of asset prices for broad asset classes, with particular attention on asset supply.

Chapter two presents a general equilibrium model that incorporates endogenous production and local housing markets, in order to explain the price relationship among human capital, housing, and stocks. Housing serves as an asset as well as a durable consumption good. The covariation of housing and stock prices can be negative if the supply of local inputs for housing production is elastic. Several examples illustrate the way the model works, for example the housing price appreciation during the economic contraction in the U.S. after 2000, and the varying degrees of stock-market participation across countries.

The model also shows that housing rent growth serves as a risk factor in the consumption-based pricing kernel, and this may mitigate the equity premium puzzle and the risk-free rate puzzle.

Chapter three examines empirically the intertemporal elasticity of substitution (IES) and static elasticity of substitution between housing and non-housing (SES), by allowing for non-homotheticity in preferences. The asset pricing implications of the second good in consumption-based models critically depend on the relative sizes of IES and SES. Estimates of SES are biased upward when moment conditions are derived under the homotheticity assumption, which is rejected empirically in this study. By allowing for non-homotheticity, we obtain lower estimates of SES ranging from 0.4 to 0.9, which are consistent with the low price elasticity of housing demand estimated in the previous literature. Estimates of IES are quite low, ranging from 0.05 to 0.14, but they might be subject to downward biases with true values of greater than one. This chapter is based on joint research with Tom Davidoff.

Chapter four focuses on the microfoundations of asset supply in the form of joint ventures. In a project jointly formed by multiple parties, strategic interactions result in a flexible arrangement of the project and delayed investments. Although keeping flexibility is optimal, given risks in the project value, one party's flexibility creates strategic uncertainty for the others, and increased uncertainty in turn encourages them to keep more flexibility as well. A positive feedback between uncertainty and flexibility leads to delayed investments. Our result makes a sharp contrast with preemptive investment to deter competitors' entry. It is also distinct from the free-rider problem since we focus on the second moment of payoffs. The model is applicable to various joint projects within a firm, across firms, and across countries.

Professor John Quigley
Dissertation Committee Chair

To Tomoko, Yuki, and Ken

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Chapter 1

Introduction

Asset prices have tremendous impacts on economic decision-making. The covariance of prices among human capital, housing, and financial assets is critical to portfolio choice, asset pricing and consumption behavior. For example, a high covariance of stock prices with other asset prices suggests that a low weight be given to stocks, given that holdings of human capital and housing are constrained at some positive levels. A low or negative covariance among the assets, in turn, stabilizes household wealth and consumption.

While substantial progress has been made in research on financial asset prices, we have a quite limited understanding of the equilibrium prices of broader asset classes. General theories of asset pricing such as the Arrow-Debreu equilibrium and the no-arbitrage pricing condition are too general to yield concrete insights into the covariance structure, while more detailed models have been either purely empirical (with a focus on a particular financial asset) or else built on simplistic assumptions regarding the production process. This dissertation contributes to the understanding of properties of asset prices for broad asset classes, with particular attention on asset supply.

In Chapter 2, I develop a general equilibrium model in order to address two questions: First, what is the covariance structure among asset prices when we incorporate endogenous responses of production sectors to technology shocks? Second, what is the role of housing in the determination of equilibrium asset prices? By relying only on straightforward economic mechanisms, I derive the direct links between primitive technology shocks and the asset price responses. I emphasize two key components: endogenous production and housing. The first component, endogenous production, characterizes asset prices and the discount factor in relation to different types of technology shocks. I analyze shocks along three dimensions: time, space and sector. The second component of the model is housing, which has at least three unique characteristics. First, housing plays a dual role as a consumption good and as an investment asset. When the utility function is not separable in housing and other consumption goods, the housing choice affects consumption and asset pricing through the discount factor. Second, housing is a durable good, which introduces an inter-temporal dependence of utility within the expected utility framework. Third, housing is a local or non-traded good. Both supply and demand of housing is supplied by combining a structure, which is capital traded nationally, and land, which is a local good. Localized housing generates important effects on the asset prices.

The first of two main results is the finding of an equilibrium relationship among asset prices for different types of technology shocks. In particular, I show that the covariation of housing prices and stock prices can be negative if the supply of local inputs for housing production (e.g., land) is elastic and vice versa. This finding is broadly consistent with the fact that the U.S. has a negative correlation and Japan has a positive correlation between these two assets. The second result is that growth of housing rent is a component of the discount factor if utility function is non-separable in housing and other goods, and thus it serves as a risk factor in the consumption-based pricing kernel. I present the possibilities that the rent growth factor mitigates the equity premium puzzle and the risk-free rate puzzle either by magnifying consumption variation or imposing a downward bias on the estimate of the inter-temporal elasticity of substitution (IES).

It turns out that whether IES is greater than static elasticity of substitution (SES) is important in asset pricing. Although there are many researches that estimate SES between durable and non-durable goods, only a few studies have estimated SES between housing and non-housing goods. A previous result based on a CES utility function is SES of 2.2 or higher, but it is not consistent with anecdotal evidence or results from previous literature on housing demand. Our conjecture is that homotheticity of preferences, implicitly built in the CES function, is a cause of the high estimate of SES because any variation in consumption share must be attributed to a price change under the homotheticity assumption.

Chapter 3, which is based on a joint research with Thomas Davidoff, examines empirically SES between housing and non-housing goods together with IES. We use aggregate time-series data, allowing for non-homothetic preferences. Our estimations reject homotheticity and give low SES between 0.4 and 0.9 when we allow for non-homotheticity. We find an upward bias in estimates of SES under the homotheticity assumption. Estimates of IES are quite low between 0.05 and 0.14, which are consistent with early studies.

Although our plain estimates indicate that IES is less than SES, eliminating downward biases in estimates of IES would reverse the sign. We also show that consumption share of housing decreases as income grows, and that it increases as more income is derived from investments. The results imply that housing demand is less elastic to income changes than non-housing, and that investors have a smaller SES than employees. Finally, we obtain additional evidence of non-homotheticity by finding significant subsistence levels of both consumption goods, based on the Stone-Geary utility function.

In Chapter 4, I build a model of asset supply. In particular, I focus on the investment decision of agents when a small number of agents form a joint investment project, in which strategic concerns play an important role. Previous studies have shown that strategic interactions typically accelerate investments for the sake of preemption and entry deterrence. However, we often observe delayed investments in joint projects. Since joint projects are now an essential basis of economic activities, it is important to understand the asset supply through joint projects.

I show that, when strategic effects exist, the equilibrium level of flexibility built in the initial contract is greater, and that the investment timing is delayed than in the competitive case. The model takes into account, not only the effect of uncertainty on the choice of flexibility, but also the reverse effect of keeping flexibility on uncertainty. Flexibility creates endogenous uncertainty through the strategic interactions among agents. While the benefit of keeping flexibility in response to the exogenous uncertainty is widely studied, the reverse channel from flexibility to uncertainty is new.

Chapter 2

Technology Shocks and Asset Price Dynamics: The Role of Housing in General Equilibrium

2.1 Introduction

Household wealth typically consists of human capital, housing, and financial assets. The covariance of prices among these broad asset classes is critical to portfolio choice, asset pricing and consumption behavior. For example, a high covariance of stock prices with other asset prices suggests that a low weight be given to stocks, given that holdings of human capital and housing are constrained at some positive levels. A low or negative covariance among the assets, in turn, stabilizes household wealth and consumption.[1]

[1]For example, it is widely believed that U.S. consumption since 2000 has been sustained in spite of depressed values of human capital and financial assets by the appreciation of housing prices.

The actual covariance structure varies across countries as well as over time. In particular, in the U.S. housing and stock prices are negatively correlated while in Japan they are positively correlated.[2] However, our theoretical understanding of the covariance structure among these broad asset classes is limited. General theories of asset pricing such as the Arrow-Debreu equilibrium and the no-arbitrage pricing condition are too general to yield concrete insights into the covariance structure, while more detailed models have been either purely empirical (with a focus on a particular financial asset) or else built on simplistic assumptions regarding the production process.[3]

In this chapter, I develop a simple general equilibrium model in order to address two questions: First, what is the covariance structure among asset prices when we incorporate endogenous responses of production sectors to technology shocks? Second, what is the role of housing in the determination of equilibrium asset prices? By relying only on straightforward economic mechanisms, I derive the direct links between primitive technology shocks and the asset price responses. Within a perfect foresight framework, asset prices in various scenarios of technology shocks are clearly shown.

The first of two main results is the finding of an equilibrium relationship among asset prices for different types of technology shocks. In particular, I show that the covariation of housing prices and stock prices can be negative if the supply of local inputs for housing production (e.g., land) is elastic and vice versa. This finding is broadly consistent with the fact that the U.S. has a negative correlation and Japan has a positive correlation be-

[2]Cocco [2000] and Flavin and Yamashita [2002] find a negative correlation in the U.S. between stock and real estate prices while Quan and Titman [1999] and Mera [2000] find a high correlation in Japan.

[3]Empirical models such as the Fama-French three factor model for equity returns are not based on complete theories. Theoretical models often reduce production processes to simply endowments (e.g. Lucas [1978]), render them implicit to the consumption process (e.g. Breeden [1979]), or posit an exogenous return/production process (e.g. Cox et al. [1985]).

tween these two assets. The result is suggestive of the housing price appreciation observed under economic contraction in the U.S. after 2000. Predictions of the model about the term structure of interest rates, the capitalization rate or "cap rate", and savings are also consistent with observations. This result also implies that an economy with inelastic land supply should exhibit either more limited stock-market participation or less homeownership because of positive covariation among asset prices. This is suggestive of the variations in stock-market participation across countries.

The second result is that growth of housing rent is a component of the discount factor if utility function is non-separable in housing and other goods, and thus it serves as a risk factor in the consumption-based pricing kernel. I present the possibilities that the rent growth factor mitigates the equity premium puzzle and the risk-free rate puzzle either by magnifying consumption variation or imposing a downward bias on the estimate of the inter-temporal elasticity of substitution (IES). The model opens an empirical opportunity to apply a new data set to the Euler equation. The risk of housing assets is also inferred from the characterization.

To derive these results, I introduce two key components: endogenous production and housing. The first component, endogenous production, characterizes asset prices and the discount factor in relation to different types of technology shocks. The discount factor is usually characterized by the consumption process without a model of endogenous production. Although real business cycle models are built on primitive technology shocks, they do not focus on asset prices but predominantly on quantity dynamics.[4] In this chapter, I

[4] A few exceptions include Rouwenhorst [1995], Jermann [1998], and Boldrin et al. [2001] who study asset price implications of technology shocks. The current model differs from theirs in several ways, including the presence of local goods. Empirically, Cochrane [1991] and Cochrane [1996] relate marginal product of

analyze shocks along three dimensions: time, space and sector. On the time dimension, there are three types of shocks: 1) current, temporary shocks, 2) anticipated, temporary shocks, and 3) current, permanent shocks. Along the space dimension, shocks can occur in the "home" city or in the "foreign" city. In the sector dimension, shocks may have an effect either on consumption-goods production or housing production.

The second component of the model is housing. Housing is the major component of the household asset holdings, but it also has, at least, three unique characteristics.[5] First, housing plays a dual role as a consumption good and as an investment asset. The portfolio choice is constrained by the consumption choice and vice versa. In particular, when the utility function is not separable in housing and other consumption goods, the housing choice affects consumption and asset pricing through the discount factor. Second, housing is a durable good, which introduces an inter-temporal dependence of utility within the expected utility framework. Inter-temporal dependence, which is also introduced via habit formation and through Epstein-Zin recursive utility, improves the performance of the asset pricing model. Third, housing is a local or non-traded good. Housing is supplied by combining a structure, which is capital traded nationally, and land, which is a local good. The demand for housing is also local since regionally distinct industrial structures generate regional variations in labor income. Localized housing generates important effects on the asset prices.[6]

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capital to discount factor.

[5] Real estate accounts for 30% of measurable consumer wealth while equity holdings, including pension and mutual funds, are only 3/5 of real estate holdings based on 2002-4 Flow of Funds Accounts of the United States. Cocco [2004] reports, using PSID, that the portfolio is comprised of 60-85% human capital, 12-22% real estate, and less than 3% stocks.

[6] The elasticity of housing supply widely varies across regions.(Green et al. [2005]) The dynamics of house price and consumption are also geographically heterogeneous. (Hess and Shin [1998] and Lustig and van Nieuwerburgh [2005])

To give a clearer idea about the economics of the model, I illustrate the mechanisms that transmit a technology shock throughout the economy. A country is composed of two cities, each of which is formed around a firm. The capital and goods markets are national while the labor, housing and land markets are local. Technology shocks have direct effects only on one city. For instance, suppose that a positive technology shock to goods-producing firms in a city raises the marginal products of capital and of labor, and hence changes interest rates and wages. The housing demand is affected by a higher lifetime income as well as a price change. The housing supply is also affected by the altered capital supply through the shifted portfolio choice. The other city, without the shock, is influenced through the national capital market. The capital supply to the foreign city is reduced due to the shifting portfolio choice across cities, and thus production and wages are reduced. Therefore, the responses of housing prices and the firms' use of capital become geographically heterogeneous. The shock also affects the next period through the inter-temporal consumption choice. The saving, or the capital supply to the next period, changes depending on the elasticity of the inter-temporal substitution. In sum, a shock has effects on the whole economy through consumption substitution between goods and between periods, and through capital substitution or portfolio selection between sectors and between cities. Different effects on the economy are analyzed for different types of technology shocks, whether temporary or permanent and whether in goods production or housing production.

The paper is organized as follows. Section 2 is a review of the related literature. In section 3 the general economic environment is specified. In section 4 the equilibrium is derived in a decentralized market institution. Section 5 includes the comparative statics and analyses of the results. Section 6 concludes and details my plan for extensions.

2.2 Related Literature

Most of models of production economies are built on the assumption of a single homogeneous good; they focus on quantities rather than asset prices. Still, a small number of recent papers introduce home production, non-tradable goods or sector-specific factors, which are all relevant in the case of housing.

In a closed economy, home production of consumption goods helps explain a high level of home investment and a high volatility of output.[7] In these models, labor substitution between home production and market production plays an important role while in the present model, capital substitution between sectors and between cities plays an important role. The housing service sector is introduced by Davis and Heathcote [2005] and two empirical regularities are explained: 1) the higher volatility of residential investment and 2) the comovement of consumption, nonresidential investment, residential investment, and GDP. They emphasize the importance of land in housing production and the effects of productivity shocks on the intermediate good sectors. However, the authors do not examine asset prices, which are the main concern here.

In an open economy, non-traded goods are introduced in the multi-sector, two-country, dynamic, stochastic, general-equilibrium (DSGE) model.[8] Non-traded goods in an open economy are comparable to local housing services and land in the current model. The important findings in this literature are that non-traded goods may help explain 1)

[7] See Greenwood and Hercowitz [1991] and Benhabib et al. [1991] among others. Boldrin et al. [2001] use a different division of production into consumption-good sector and investment-good sectors.

[8] See Tesar [1993], Stockman and Tesar [1995] and Lewis [1996] among others.

the high correlation between savings and investment, 2) the low cross-country correlation of consumption growth, and 3) home bias in investment portfolio. Again, price dynamics are not considered in this literature.

The asset pricing literature typically relies on a single good by implicitly assuming the separability of the utility function.[9] Accordingly, most empirical works put little emphasis on housing as a good, relying on a single category of good defined in terms of non-durable goods and services.[10] Housing is often taken into account in the portfolio choice problem in partial equilibrium.[11] Incorporating the high adjustment cost of housing leads to interesting results such as high risk aversion and limited stock-market participation. However, the implications of the analyses are limited in scope since covariance structures of returns are exogenously given. Others examine the lifecycle profiles of the optimal portfolio and consumption when housing is introduced.[12] These works are complementary to the research reported in this chapter since they address non-asset pricing issues in general equilibrium.

Only a few papers examine the effects of housing on asset prices. Piazzesi et al. [2004] start from the Euler equation and examine the stochastic discount factor (SDF) when the intra-period utility function has a constant elasticity of substitution (CES) form, which is non-separable in consumption goods and housing services. They show that the ratio of housing expenditure to other consumption, which they call composition risk, appears in the

[9] See for example Lucas [1978], Breeden [1979], Cox et al. [1985], Rouwenhorst [1995] and Jermann [1998].

[10] Exceptions include Dunn and Singleton [1986], Pakos [2003] and Yogo [2005], who take account of durable consumption. However, their durable consumption ignores housing in favor of motor vehicles, furniture, appliances, jewelry and watches.

[11] The demand for housing or mortgages are considered by Henderson and Ioannides [1983], Cocco [2000], Sinai and Souleles [2004], Cocco and Campbell [2004], and Shore and Sinai [2004]. The effects of housing on the portfolio of financial assets are considered by Brueckner [1997], Flavin and Yamashita [2002], Cocco [2004], and Chetty and Szeidl [2004] among others.

[12] See for example, Ortalo-Magne and Rady [2005], Platania and Schlagenauf [2000], Cocco et al. [2005a], Fernandez-Villaverde and Krueger [2003], Li and Yao [2005], and Yao and Zhang [2005].

SDF. They then proceed to conduct an empirical study taking the observed consumption process as the outcome of a general equilibrium. Two key differences from the present model are 1) they do not include the link with technologies, and 2) their housing is not distinct from other durable goods. Lustig and van Nieuwerburgh [2004a] focus on the collateralizability of housing in an endowment economy. They use the ratio of housing wealth to human capital as indicating the tightness of solvency constraints and explaining the conditional and cross-sectional variation in risk premia. Their result is complementary to those reported below, as they show that another unique feature of housing, collateralizability, is important in asset pricing. Kan et al. [2004], using a DSGE model, show that the volatility of commercial property prices is higher than residential property prices and that commercial property prices are positively correlated with the price of residential property. Although housing is distinguished from commercial properties, its locality is not considered. In addition, their focus is also not on asset pricing in general but is limited to property prices.

2.3 The Model

2.3.1 Technologies

There are two goods: a composite good (Y_t) and housing services (H_t). The latter is a quality-adjusted service flow; larger service flows are derived either from a larger house or from a higher quality house.

Composite goods are produced by combining business capital (K_t) and labor (L_t) while housing services are produced by combining housing structures (S_t) and land (T_t).[13]

[13]The land should be interpreted as the combination of non-structural local inputs. In particular, it includes all local amenities raising the quality of housing service, such as parks. The land supply function

The production functions are both Cobb-Douglas:

$$Y_t = Y(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{2.1a}$$

$$H_t = H(B_t, S_t, T_t) = B_t S_t^{\gamma} T_t^{1-\gamma}, \tag{2.1b}$$

where A_t and B_t are total factor productivities of goods and housing production, respectively.[14] Parameters α and γ are the share of capital cost in the outputs of composite goods and housing services, respectively.[15]

The production functions exhibit a diminishing marginal product of capital (MPK) so that the return depends on production scale, unlike in the linear technology case. This property, together with changing productivities, allows the return to vary over time and across states. Note also that a technology shock to housing production can be interpreted as a preference shock in the current model. This is because produced housing services directly enter into the utility function. A higher B_t could be interpreted as implying that a greater utility is derived from the same level of structures and land and that the households are less willing to pay for housing due to their reduced marginal utility.

2.3.2 Resource Constraint

Composite goods are used either for consumption or investment. The resource constraint is

$$Y_t = C_t + I_t + J_t, \tag{2.2}$$

is explained in the household section.

[14]With the Cobb-Douglas production function, a total factor productivity shock can be described in terms of a shock to the capital-augmenting technology or as one to the labor-augmenting technology. For example, we can rewrite the production function as $Y = AK^{\alpha}L^{1-\alpha} = \left(A^{1/\alpha}K\right)^{\alpha} L^{1-\alpha} = K^{\alpha}\left(A^{1/(1-\alpha)}L\right)^{1-\alpha}$.

[15]These parameters also represent the elasticity of output with respect to capital in the Cobb-Douglas production function.

where C_t is consumption, I_t and J_t the investment in business capital and housing structures, respectively. The equations defining the accumulation of business capital and housing structures are

$$K_{t+1} = (1 - \delta_K) K_t + I_t, \tag{2.3a}$$

$$S_{t+1} = (1 - \delta_S) S_t + J_t, \tag{2.3b}$$

where δ_K and δ_S are the constant depreciation rate of business capital and housing structures, respectively. I assume $\delta_K = \delta_S = \delta$ for simplicity.

Note that the inclusion of the housing structures makes housing services a durable good. Consumption of housing services is directly linked with the accumulated structures while the amount of the composite goods consumption is chosen under the constraint (2.2). This makes housing services different from other goods.

2.3.3 Preferences

Consumers' preferences are expressed by the following expected utility function:

$$U = E_0 \left[\sum_{t=1}^{\infty} \beta^t u(C_t,\ H_t) \right] \tag{2.4}$$

where E_0 is the conditional expectation operator given the information available at time 0, β is the subjective discount factor per period, $u(\bullet)$ is the intra-period utility function over composite goods (C_t) and housing services (H_t). In a two period model with perfect foresight, the lifetime utility becomes

$$U = u(C_1, H_1) + \beta u(C_2, H_2).$$

The CES-CRRA (constant relative risk aversion) intra-period utility function is adopted:

$$u(C_t, H_t) = \frac{1}{1-\frac{1}{\theta}} \left(C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}} \right)^{\left(1-\frac{1}{\theta}\right)/\left(1-\frac{1}{\rho}\right)}, \tag{2.5}$$

where $\rho > 0$ is the elasticity of intra-temporal substitution between composite goods and housing services, and $\theta > 0$ is the parameter for the elasticity of inter-temporal substitution. The simplest special case is that of separable log utility, $u(C_t, H_t) = \ln C_t + \ln H_t$, which corresponds to $\rho = \theta = 1$.

The non-separability between composite goods and durable housing in the CES specification delinks the tight relationship between the relative risk aversion and the elasticity of inter-temporal substitution. Even though the lifetime utility function has a time-additive expected utility form, the durability of housing makes the utility function intertemporally dependent.[16]

Other specifications that also break the link between relative risk aversion and IES include habit formation and Epstein-Zin recursive utility. Habit formation is similar to durable consumption, but past consumption in the habit-formation model makes the agent less satisfied while past expenditure on durables makes the agent more satisfied. Both habit formation and Epstein-Zin recursive utility are known to resolve partially the equity premium puzzle.

With the non-separability of the CES function, the relative risk aversion is not simply $1/\theta$; it is defined as the curvature of the value function, which depends on durable

[16] It might seem that the utility is not specified over housing as a durable but as contemporaneous housing services produced by real estate firms. However, housing services depend on the real estate firms' past investments in the housing structure, which are analogous to the households' expenditure on durable housing. Indeed, "real estate firms" can be characterized as the internal accounts of households. These "real estate firms" are set up just to derive explicitly the housing rent.

housing. CRRA utility over a single good is a special case in which the curvature of the value function coincides with the curvature of utility function. Note also that the elasticity of inter-temporal substitution for composite goods in the continuous-time limit is defined as the weighted harmonic mean of ρ and θ:

$$\begin{aligned} IES &= \left[-C_t \frac{u_{CC}(C_t, H_t)}{u_C(C_t, H_t)}\right]^{-1} \\ &= \left[\frac{1}{\rho}\left(1 - \frac{C_t^{1-\frac{1}{\rho}}}{C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}}\right) + \frac{1}{\theta}\left(\frac{C_t^{1-\frac{1}{\rho}}}{C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}}\right)\right]^{-1}, \end{aligned}$$

where the weight is the share of the composite goods component in the aggregator.[17]

2.3.4 Cities

There are two cities of the same initial size, in each of which households, goods-producing firms, and real estate firms operate competitively. The variables and parameters of the city with technology shocks ("home" city) with plain characters (C_t, etc.) and those of the other ("foreign") city with starred characters (C_t^*, etc.).

Each "city" should not be interpreted literally. Instead, a "city" is understood to be a set of cities or regions that share common characteristics in their industrial structure and land supply conditions. For example, a technology shock to the IT industry mainly affects the cities whose main industry is the IT industry. A "city" in this chapter represents the collection of such cities that are affected by the same technology shock.

[17] See Deaton [2002] and Flavin and Nakagawa [2004] for detailed discussions on the delinking of EIS and risk aversion. Yogo [2005] shows the importance of non-separability between durables and non-durables in explaining the equity premium. Limitations caused by homotheticity induced by the CES form are discussed in Pakos [2003].

2.3.5 Discount Factors and Asset Prices

Let $\phi_{t,t+1}$ denote the discount factor for time $t+1$ as of time t. The price of any asset is expressed as the expected return in units of the numeraire multiplied by the discount factor. For example, the ex-dividend equity price of firm f, $e_{f,t}$, is expressed in terms of the dividend stream $D_{f,t}$ and the discount factor as

$$e_{f,t} = E_t \left[\sum_{j=1}^{\infty} \phi_{t,t+j} D_{f,t+j} \right].$$

The importance of the covariance between the discount factor and the return can be seen from this equation. The j-period risk-free discount factor as of time t (i.e. the price of a bond that will deliver one dollar j periods later without fail) is $E_t \left[\phi_{t,t+j} \right]$. Equivalently, using the j-period risk-free rate of return, $i_{j,t}$,

$$\frac{1}{i_{j,t}} = E_t \left[\phi_{t,t+j} \right].$$

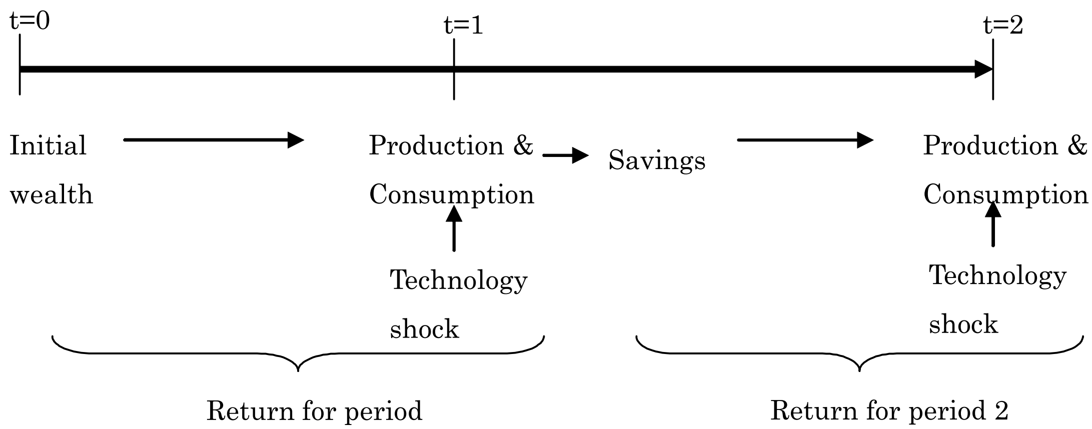
Without uncertainty, the relationship in expectation becomes the exact relationship:

$$\begin{aligned} e_{f,t} &= \sum_{j=1}^{\infty} \phi_{t,t+j} D_{f,t+j}, \\ \frac{1}{i_{j,t}} &= \phi_{t,t+j}. \end{aligned}$$

2.4 Market Institutions and Equilibrium in a Two-Period Model With Perfect Foresight

I derive the decentralized market equilibrium in a two-period model with perfect foresight. Figure 2.1 presents the time line of economic activities.

Figure 2.1: Time line.



(Goods-producing firm) Goods-producing firms competitively produce composite goods by combining capital and labor. Each goods-producing firm in the home city solves the following problem in each period, taking as given interest rates (i_1, i_2), wages (w_1, w_2) and total factor productivities (A_1, A_2). The firms in the foreign city solve the identical problem with possibly different variables and parameters.

$$\max_{K_t, L_t} Y(A_t, K_t, L_t) - (i_t - 1 + \delta) K_t - w_t L_t, \quad t = 1, 2.$$

This objective function is a reduced form in which the firm's capital investment decision does not explicitly show up and in which the firm only recognizes the periodic capital cost. (This simplification is possible because there is no stock adjustment cost.)

The first-order conditions define the factor demands of the goods-producing firm:

$$K_t \quad : \quad i_t - 1 + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha}, \tag{2.6a}$$

$$L_t \quad : \quad w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{2.6b}$$

As usual, the interest rate is equal to $1-\delta$ plus the marginal product of capital (MPK), and the wage is equal to the marginal product of labor. In equilibrium with perfect foresight, the national market for capital implies that capital allocations are adjusted until the interest rates are equated across sectors and cities. Wages are unique to the city since the labor market is local.

(Real estate firm) Real estate firms produce housing services by combining land and structures. Each real estate firm solves the following problem in each period, taking as given the housing rent (p_1, p_2), the interest rate (i_1, i_2), the land rent (r_1, r_2) and the total factor productivity (B_1, B_2). (The firms in the foreign city solve identical problems with starred variables.)

$$\max_{S_t, T_t} p_t H(B_t, S_t, T_t) - (i_t - 1 + \delta) S_t - r_t L_t, \quad t = 1, 2.$$

As noted, these "real estate firms" can be also interpreted as the internal accounts of households since homeowners are not distinguished from renters. Nevertheless, I prefer describing the real estate industry in order to obtain explicitly the housing rent.

The first-order conditions define the factor demands of housing production:

$$S_t \quad : \quad i_t - 1 + \delta = p_t \frac{\partial H_t}{\partial S_t} = \gamma B_t p_t \left(\frac{T_t}{S_t}\right)^{1-\gamma}, \qquad (2.7a)$$

$$T_t \quad : \quad r_t = p_t \frac{\partial H_t}{\partial T_t} = (1-\gamma) B_t p_t \left(\frac{S_t}{T_t}\right)^{\gamma}. \qquad (2.7b)$$

The interest rate and the land rent are equal to the marginal housing product of structure (MHPS) and of land (MHPL), respectively, in units of the numeraire. Again, the interest rate will be equated across sectors and cities in equilibrium while the land rent is locally determined.

(Households) Households are endowed with initial wealth (W_0) and land. They provide capital, land and labor in each period to earn financial, land and labor income, respectively, and spend income on consumption of composite goods, housing services, and savings (W_1). The savings can be freely allocated among sectors and cities.

Labor is inelastically supplied and normalized at one. Households are assumed to be immobile across cities. This assumption is reasonable since most of the population does not migrate across regions. The immobility of labor will result in wage differentials across cities. The free mobility of households would make labor more like capital and render the production function linear in inputs. The costs of capital and labor would be equated across cities and the price responses would become more moderate. While the mobility would generate more moderate results on the asset price, it would not greatly change the overall results as long as homothetic CES preferences are maintained.[18]

Land supply is assumed to be iso-elastic:

$$T_t = r_t^{\mu}, \quad t = 1, 2,$$

where μ is the price elasticity of supply. $\mu = 0$ represents a perfectly inelastic land supply at one and $\mu = \infty$ represents perfectly elastic land supply. By this simple form, land supply elasticity and asset prices are linked in a straightforward way. While the land supply is obviously constrained by the topographic conditions of the city, other conditions such as zoning regulations and current population densities are also critical. For example, the infill development and the conversion from agricultural to residential use make the land supply elastic. The elasticity can also be understood as reflecting short-run and long-run elasticities.

[18]With CES preferences, the income elasticity of housing demand is one. Therefore, even if the housing demand per household is altered by the wage income, the offsetting change in the population will limit the effects on total housing demand.

For example, if eminent domain is politically hard to use in providing a local amenity or if the current landlords rarely agree on redevelopments, the housing supply process may take longer than a business cycle, in which case the land supply is more inelastic.[19]

Each household solves the following problem, taking as given the housing rents, land rents, interest rates and wages.

$$\max_{\{C_t,H_t\}} u\left(C_1,H_1\right)+\beta u\left(C_2,H_2\right)$$

$$s.t.\ C_1+p_1H_1+W_1=i_1W_0+r_1T_1+w_1$$

$$C_2+p_2H_2=i_2W_1+r_2T_2+w_2.$$

The above dynamic budget constraints can be rewritten as the lifetime budget constraint:

$$\begin{aligned} C_1+p_1H_1+\frac{1}{i_2}\left(C_2+p_2H_2\right) &= i_1W_0+r_1T_1+w_1+\frac{1}{i_2}\left(r_2T_2+w_2\right) \\ &\equiv Inc. \end{aligned}$$

The RHS of the lifetime budget constraint is defined as the lifetime income, Inc.

The first-order conditions for the CES-CRRA utility are[20]

$$p_t^{\rho}H_t = C_t, \tag{2.8a}$$

$$\begin{aligned} i_2 &= \left(\beta\frac{\partial u/\partial C_2}{\partial u/\partial C_1}\right)^{-1} \\ &= \frac{1}{\beta}\left(\frac{C_2}{C_1}\right)^{\frac{1}{\theta}}\left[\frac{1+\left(H_2/C_2\right)^{\frac{1}{1-1/\rho}}}{1+\left(H_1/C_1\right)^{\frac{1}{1-1/\rho}}}\right]^{\frac{\theta-\rho}{\theta(1-\rho)}}. \end{aligned} \tag{2.8b}$$

The interest rate is the reciprocal of the inter-temporal marginal rate of substitution (IMRS). That is, the IMRS is the discount factor in this economy. In the log utility case,

[19] Many development projects in Japan take more than twenty years to complete. This is an example of an inelastic supply due to the slow development process.

[20] In the log-utility case, they reduce to $p_tH_t = C_t$ and $i_2 = \left(\beta\ \frac{\partial u}{\partial C_2}\Big/\frac{\partial u}{\partial C_1}\right)^{-1} = \left(1/\beta\right)\left(C_2/C_1\right)$.

the interest rate is proportional to consumption growth because of the unit elasticity of inter-temporal substitution. The inter-temporal consumption substitution expressed by this Euler equation, together with the intra-temporal substitution between two goods, is a key driver of the economy. The IMRS is discussed, in a greater detail, in the next section since it is a key to understanding the economy.

With the lifetime budget constraint, I obtain the consumption demands:[21]

$$C_1 = \left(1+p_1^{1-\rho}\right)^{-1} \left\{ 1 + \beta^{\theta} i_2^{-(1-\theta)} \left(\frac{1+p_2^{1-\rho}}{1+p_1^{1-\rho}} \right)^{\frac{1-\theta}{1-\rho}} \right\}^{-1} Inc, \tag{2.9a}$$

$$C_2 = \beta^{\theta} i_2^{\theta} \left(\frac{1+p_2^{1-\rho}}{1+p_1^{1-\rho}} \right)^{\frac{\rho-\theta}{1-\rho}} C_1, \tag{2.9b}$$

$$H_t^{dem} = \frac{C_t}{p_t^{\rho}}. \tag{2.9c}$$

Note that the housing rents have no effect on the consumption demand in the log utility case while they *do* have an effect on it in general. It is also clear that the expenditure ratio of housing, $p_t H_t / C_t$, is always 1 in the log case while it is $p_t^{1-\rho}$ in general.

2.4.1 Discount Factors and the Role of Rent Growth

Before solving for a general equilibrium, I provide a new way of characterizing the discount factor and describe the link between rent growth and asset pricing. Small manipulations to (2.6a), (2.7a), and (2.8a) yield three different ways of expressing the

[21] In the log-utility case, they reduce to $C_1 = Inc/\left[2\left(1+\beta\right)\right], C_2 = \beta i_2 C_1$, and $H_t^{dem} = C_t/p_t$.

discount factor:[22]

$$\phi_{1,2} = \left[1 + \frac{\partial Y_2}{\partial K_2} - \delta\right]^{-1} \qquad \text{(Reciprocal of MPK)} \tag{2.10a}$$

$$= \left[1 + p_2 \frac{\partial H_2}{\partial S_2} - \delta\right]^{-1} \qquad \text{(Reciprocal of MHPS)} \tag{2.10b}$$

$$= \beta \left(\frac{C_2}{C_1}\right)^{-\frac{1}{\theta}} \left[\frac{1 + p_2 H_2/C_2}{1 + p_1 H_1/C_1}\right]^{\frac{\rho-\theta}{\theta(1-\rho)}} \quad \text{(IMRS).} \tag{2.10c}$$

These relationships hold for the foreign city as well. Indeed, the discount factor is the center piece that is common to all agents in the economy. The first equation (2.10a), which is empirically exploited by Cochrane [1991], is used to understand the effect of goods-sector shocks. The second equations (2.10b) are useful when considering housing shocks. The third equation (2.10c) includes the expenditure share of housing consumption, which Piazzesi et al. [2004] call the composition risk and empirically exploit.

I derive a different formula that includes only the housing rents in the second term by using (2.9a), (2.9b), and (2.9c).

$$\phi_{1,2} = \beta \left\{ \left(\frac{C_2}{C_1}\right) \left[\frac{\left(1 + p_2^{1-\rho}\right)^{\frac{1}{1-\rho}}}{\left(1 + p_1^{1-\rho}\right)^{\frac{1}{1-\rho}}}\right]^{\theta-\rho} \right\}^{-\frac{1}{\theta}}$$

$$\equiv \beta \left\{ g_{c,2} \cdot g_{p,2}{}^{\theta-\rho} \right\}^{-\frac{1}{\theta}}, \tag{2.11}$$

where $g_{c,2} \equiv C_2/C_1$ is the consumption growth, and $g_{p,2} \equiv \left(1 + p_2^{1-\rho}\right)^{\frac{1}{1-\rho}} \Big/ \left(1 + p_1^{1-\rho}\right)^{\frac{1}{1-\rho}}$ is the growth of the CES-aggregated price index. Note that $g_{p,2}$ is a monotonically increasing function of the rent growth. A high $g_{p,2}$ means that the numeraire good in period 2 is relatively abundant and cheap, or equivalently that housing is relatively precious and expensive.

[22] For the log utility, IMRS reduces to $\phi_{1,2} = \beta \left(C_2/C_1\right)^{-1}$.

This equation gives a new insight about the meaning of rent growth in the context of the asset pricing. The IMRS ($\phi_{1,2}$), which measures how "under-satisfied" the household is in the second period, basically has the same form as in the single good CRRA case. When consumption growth is high, the household is more satisfied in that period and the marginal utility is lower. However, the level of satisfaction is not simply measured by consumption growth but by the consumption growth augmented by the growth of the aggregate price $g_{p,2}$ raised to the $\theta - \rho$.

When the two goods are relatively substitutable ($\rho > \theta$), a high growth of the aggregate price $g_{p,2}$ (i.e., abundant composite goods) reduces the satisfaction gained from composite goods because of their abundance. A low $g_{p,2}$, on the contrary, raises satisfaction from consuming composite goods because they are more precious. Similarly, when the two goods are relatively complementary ($\rho < \theta$), a low $g_{p,2}$ (i.e., abundant housing) increases the need for composite goods. The consumption growth is adjusted downward. Conversely, a high $g_{p,2}$ (i.e., precious housing) makes composite goods less needed, so that consumption growth is adjusted upward.

In sum, the housing rent measures the relative abundance of composite goods. This abundance affects the marginal utility of composite goods differently depending on the relative substitutability between the goods.

Proposition 1 *Housing rent growth, measured by the growth of the CES-aggregated price index ($g_{p,2}$), is a component of the discount factor if utility function is non-separable in housing and the numeraire good. The sign of the relationship between rent growth and the* *discount factor is determined by relative substitutability between the two goods:*

$$\frac{\partial \phi_{1,2}}{\partial g_{p,2}} > 0 \ (< 0) \quad \text{for } \rho > \theta \ (\rho < \theta). \tag{2.12}$$

This intuition is also confirmed by examining cross-partial derivative of the intra-period utility function: $sgn\left(\partial^2 u\left(C_t, H_t\right) / \partial H_t \partial C_t\right) = sgn\left(\theta - \rho\right)$.[23] Abundant housing raises the marginal utility of consumption when $\theta > \rho$. The consumption growth, however, is not independent of the rent growth. Therefore, another partial equilibrium argument leads to the following corollary.

Corollary 2 *If the discount factor is fixed at a given level, rent growth and consumption growth have a positive (negative) relationship when the two goods are relatively substitutable (complementary):*

$$\left.\frac{\partial g_{c,2}}{\partial g_{p,2}}\right|_{d\phi_{1,2}=0} > 0 \ (< 0) \quad \text{for } \rho > \theta \ (\rho < \theta).$$

The above arguments, however, are all based on the partial derivatives of the discount factor. The discount factor, the rent growth, and the consumption growth are determined in general equilibrium and their changes cannot be identified merely with reference to the Euler equation. Indeed, I show that the relationship between the consumption growth and the discount factor change signs depending on parameter values and the type of shock involved. The equilibrium responses to a technology shock, which will be discussed after defining the equilibrium, provide a fresh look at several related results: Tesar [1993]

[23] Specifically, the cross-partial derivative is

$$\frac{\partial^2 u\left(C_t, H_t\right)}{\partial H_t \partial C_t} = \left(\frac{\theta - \rho}{\rho\theta}\right)\left(C_t H_t\right)^{-\frac{1}{\rho}}\left[C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}\right]^{\left(1-\frac{1}{\theta}\right)/\left(1-\frac{1}{\rho}\right)-2}.$$

who considers an endowment shock to the non-tradables; and Piazzesi et al. [2004] who consider the relationship between the discount factor and the expenditure share of housing. In particular, it is shown that the rent growth component may mitigate the equity premium puzzle and the risk-free rate puzzle. The characterization of the discount factor using housing rent provides an opportunity to use different data sets in empirical analyses.[24]

2.4.2 Definition of the Equilibrium

Markets are for composite goods, housing services, land, labor, and capital. Walras' law guarantees market clearing in the goods market, and the market-clearing conditions are imposed for the other markets. The multi-sector structure necessitates a numerical solution. Detailed derivation of the equilibrium is shown in Appendix A.

Definition 3 ***A competitive equilibrium*** *in this 2-period, 2-city economy with perfect foresight is the allocation* $\{C_t,\ C_t^*,\ H_t,\ H_t^*,\ W_1,\ W_1^*,\ Y_t,\ Y_t^*,\ K_t,\ K_t^*,\ L_t,\ L_t^*,\ S_t,\ S_t^*,\ T_t,\ T_t^*\}_{t=1,2}$ *and the prices* $\{p_t,\ p_t^*,\ w_t,\ w_t^*,\ i_t,\ r_t,\ r_t^*\}_{t=1,2}$ *such that*

1. optimality is achieved for households, goods-producing firms, and real estate firms, and

2. all market-clearing conditions and resource constraints are met.

2.5 Comparative Statics

The goals are to understand 1) the observed dynamic relationship among various asset classes, 2) the relationship between asset prices and business cycles, and 3) the role of housing in the economy. Different types of technology shocks are introduced as follows.

[24] Housing rent data have several advantages over housing consumption data in terms of their availability and accuracy.

	Goods Production	Housing Production
Temporary, current	$\Delta A_1 \gtrless 0$	$\Delta B_1 \gtrless 0$
Temporary, anticipated	$\Delta A_2 \gtrless 0$	$\Delta B_2 \gtrless 0$
Permanent, current	$\Delta A_1 = \Delta A_2 \gtrless 0$	$\Delta B_1 = \Delta B_2 \gtrless 0$

Technology shocks are given to the home city. Different parameter values are allowed for

μ : Elasticity of land supply
ρ : Elasticity of intra-temporal substitution between C and H
θ : Parameter for inter-temporal substitution.

2.5.1 Mitigating the Equity Premium Puzzle and the Risk-Free Rate Puzzle

The equity premium puzzle is the fact that the historical risk premium associated with equity is too high to be explained by the observed covariance between the consumption-based discount factor and the return under plausible levels of risk aversion. Since the puzzle arises from too little variation in the consumption growth, any factor that magnifies the variation of the consumption growth in the Euler equation helps to resolve the puzzle. A closely related issue is a low estimate of IES, since the coefficient of relative risk aversion is the reciprocal of IES with a single good power utility specification. A low IES implies a much higher interest rate than the historical level. This is called the risk-free rate puzzle. Previous estimates of IES are typically quite low and even negative.

The model generates an equilibrium relationship among consumption growth, rent growth, and the discount factor. There are two cases in which these puzzles are mitigated.

Proposition 4 *The equity premium puzzle and the risk-free rate puzzle are mitigated in the following two cases.*

Case 1: *Magnified variation of consumption growth in response to anticipated shocks to goods production when $\theta > \rho$.*
Case 2: *Estimates of IES are biased downward by shocks to housing production.*

Case 1 is based on a positive covariation of consumption growth ($g_{c,2}$) with the rent growth factor (${g_{p,2}}^{\theta-\rho}$) in (2.11). Figure 2.2, Panel A presents the variation of augmented consumption growth ($g_{c,2} \cdot {g_{p,2}}^{\theta-\rho}$) and its components in response to anticipated shocks to goods production (ΔA_2) when $\rho = 0.2$ and $\theta = 1.8$. Augmented consumption growth exhibits much greater variation than plain consumption growth since the rent growth factor changes in the same direction. The covariation of $g_{c,2}$ and ${g_{p,2}}^{\theta-\rho}$ has a positive sign when the two goods are relatively complementary ($\theta > \rho$). (Panel B) Suppose that a positive future shock to goods production is anticipated. ($\Delta A_2 > 0$) Both consumption (C_2) and rent (p_2) increase in the future, which drives both consumption growth ($g_{c,2}$) and rent growth higher. The rent growth factor also increases if $\theta - \rho$ is positive and vice versa. With other types of shocks, the covariation is mainly negative and variation of consumption growth is dampened. Therefore, this case applies if asset prices are mainly driven by news about future productivity shocks, and if the two goods are relatively complementary.

The condition $\theta > \rho$ is not unrealistic although previous estimates of the elasticities of substitution are mixed. Regarding intra-temporal substitution (ρ), most studies define durables as motor vehicles, furniture, jewelry and so on. The estimates of ρ for these goods range from 0.4 to 1.2. A smaller number of studies include housing, whose estimates range from 0.2 to 2.2.[25] The estimates of IES (θ) are also mixed: Although a quite low IES (close to zero or even negative) is usually estimated, much higher estimates (from 1 to 3) are also presented.[26]

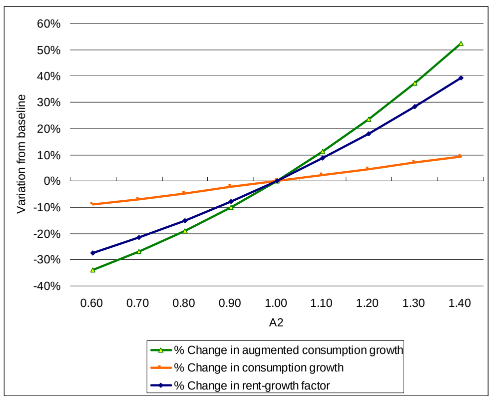
Case 2 explains a bias arising from a mis-specification. In equilibrium, housing shocks lead to positive covariation of the discount factor and consumption growth. (Panel C) If a model of a single good power utility, $\phi_{1,2} = \beta\, {g_{c,2}}^{-1/\theta}$, is applied to this situation, the positive covariation results in a negative estimate of θ since θ is estimated by ignoring the rent growth factor (${g_{p,2}}^{\theta-\rho}$) in (2.11). Therefore, if housing shocks are mixed with goods production shocks, the estimate of θ is biased downward. This implies an ambiguous relationship between consumption growth and the discount factor, which in turn cautions us not to make an immediate inference about the discount factor by looking only at the consumption growth.

The discount factor ($\phi_{1,2}$) and consumption growth ($g_{c,2}$) move in the same direction since both of them move inversely with the rent growth factor, which sharply responds to housing shocks. (Panel D) The inverse relationship between consumption growth and the rent growth factor is generated as follows. Suppose a positive housing shock occurs at $t = 2$ ($\Delta B_2 > 0$). Rent growth declines and the rent growth factor also declines when $\theta > \rho$ and vice versa. On the other hand, the consumption at $t = 2$, and thus consumption growth increases when $\theta > \rho$ because of the complementarity of the two goods. An analogous mechanism works with ΔB_1.

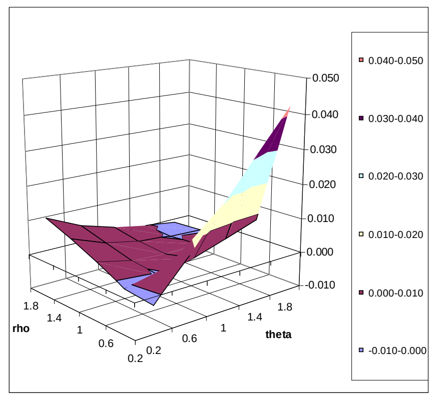
[25] See Tesar [1993] and Yogo [2005] for non-housing durables, and Lustig and van Nieuwerburgh [2004a], Piazzesi et al. [2004], and Davis and Martin [2005] for housing.

[26] Among the large body of literature on the EIS estimation, Hall [1988] finds it to be negative and Yogo [2005] estimates it at 0.02 while Vissing-Jorgensen and Attanasio [2003] find it between 1 and 2 and Bansal and Yaron [2004] estimate the EIS between 1.9 and 2.7.

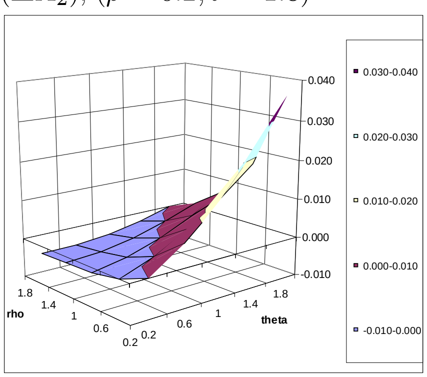
Figure 2.2: Mitigating the equity premium puzzle and the risk-free rate puzzle.



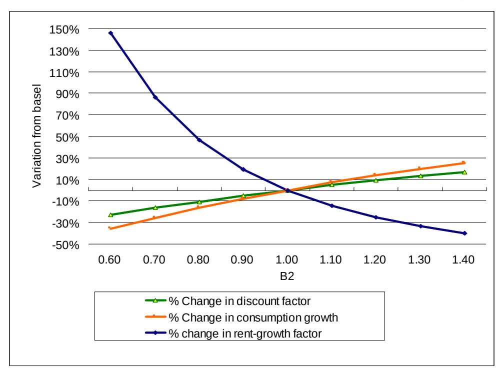
Panel A: Variation of consumption growth to an anticipated shock to goods production (ΔA_2), ($\rho = 0.2, \theta = 1.8$)



Panel C: Covariation of consumption growth ($g_{c,2}$) and the discount factor ($\phi_{1,2}$) to an anticipated shock to housing production (ΔB_2)



Panel B: Covariation of consumption growth ($g_{c,2}$) and rent growth factor ($g_{p,2}^{\theta-\rho}$) to an anticipated shock to goods production ($\Delta A2$)



Panel D: Variation of the discount factor, consumption growth, and rent growth factor to an anticipated shock to housing production (ΔB_2), ($\rho = 0.2, \theta = 1.8$)

Figure 2.2 presents two cases in which the equity premium puzzle and the risk-free rate puzzle are mitigated. Panels A and D present percentage changes from their baselines against A_2. Panels B and C show covariation of $g_{c,2}$ with $g_{p,2}^{\theta-\rho}$ and $g_{c,2}$ with $\phi_{1,2}$, respectively, against ρ and θ.

2.5.2 Effects on the Discount Factor

Figure 2.3 presents selected comparative statics of the discount factor. They serve as the basis for understanding the asset price relationship. With a positive goods production shock ($\Delta A_t > 0$), the marginal product of capital becomes higher at any level of capital. The equilibrium interest rate (i_t) rises, or equivalently, the discount factor ($\phi_{t-1,t}$) falls although more capital (K_t) is allocated from the foreign city. These effects hold regardless of parameters. (Panel A) The discount factor in the other period is also affected via savings, as an increase in the lifetime income motivates households to smooth consumption by adjusting their savings (W_1). With $\Delta A_1 > 0$, the savings at $t = 1$ (capital supply for $t = 2$) are raised and i_2 falls ($\phi_{1,2}$ rises). (Panel B) With $\Delta A_2 > 0$, the reduced savings at $t = 1$ allow a greater demand for goods at $t = 1$ and generally raise i_1 (lowers $\phi_{0,1}$) although the effects are much smaller due to the fixed capital supply.

If a positive shock is given to housing production ($\Delta B_t > 0$), the effects are much smaller. Although housing production (H_t) increases, expenditures ($p_t H_t$) are less affected since the rent (p_t) decreases. The marginal housing product may even fall if the housing rent falls enough. The effects on the contemporaneous discount factor depend on the rate of substitution between the goods. If the intra-temporal substitution (ρ) is low (i.e. the two goods are complements), the contemporaneous discount factor ($\phi_{t-1,t}$) rises.[27] The reason is as follows. A low intra-temporal substitution means a low price elasticity of housing demand. The increased housing consumption necessitates a much greater reduction in housing rent (p_t) so that the housing expenditure ($p_t H_t$) decreases. The marginal housing product of

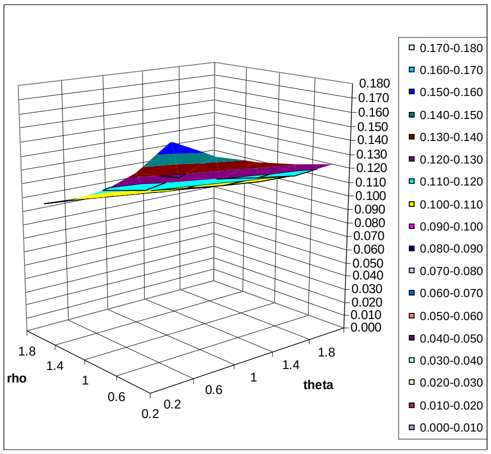
[27] To be precise, θ also has a secondary effect on $\phi_{0,1}$ since the inter-temporal substitution affects capital demand. The effect of θ is more apparent when the shock is temporary.

structure also falls, which means that the discount factor rises. If the substitution is high, the opposite is true and the discount factor falls. With the log utility, ΔB_t has no effect on the discount factor. (Panel C)

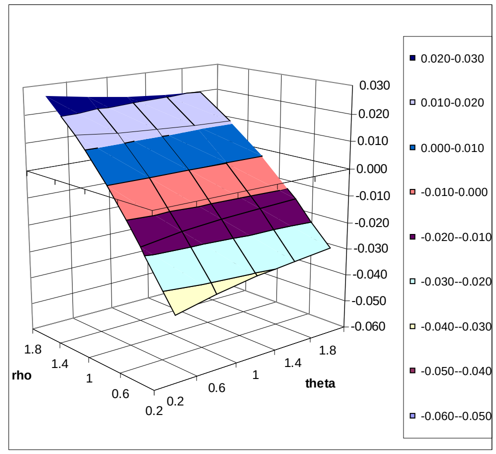
The other period is again affected through inter-temporal substitution. Since the effects on lifetime income are quite small, the inter-temporal substitution rather than the consumption smoothing may come into play if θ is large. Consider $\Delta B_1 > 0$. (Panel D) As θ becomes large, future resources are shifted toward the current period as savings are reduced. This raises i_2. If θ is small, the savings are increased (for consumption smoothing) and i_2 falls.[28] With $\Delta B_2 > 0$, the same mechanism affects savings although the effects on $\phi_{0,1}$ are small due to the fixed capital supply. As θ becomes large, the current capital demand is reduced by the increased savings, and i_1 falls. The general equilibrium effects on the discount factor are summarized in Table 2.1.

[28] To be precise, ρ has a secondary effect on $\phi_{1,2}$ since intra-temporal substitution affects capital demand.

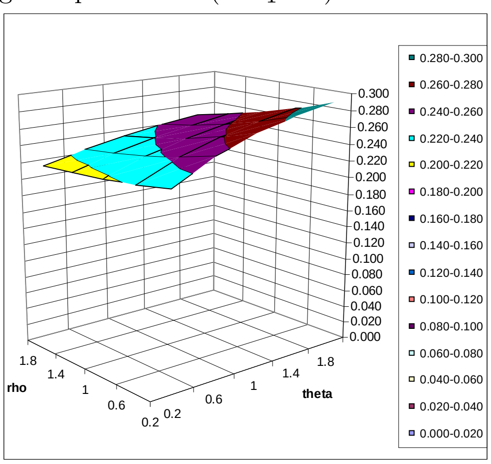
Figure 2.3: Effects of a shock on the discount rate.



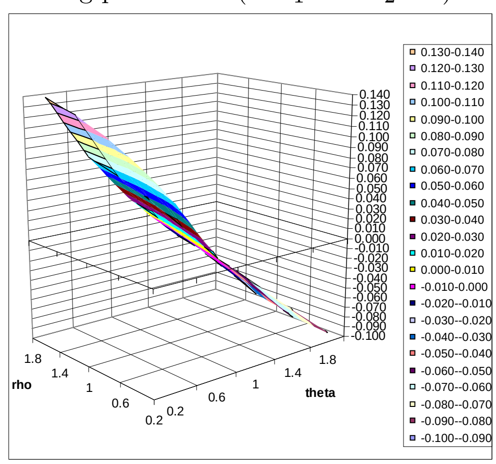
Panel A: Response of interest rate ($\Delta i_1 = 1/\Delta\phi_{0,1}$) to a positive shock to goods production ($\Delta A_1 > 0$).



Panel C: Response of interest rate ($\Delta i_1 = 1/\Delta\phi_{0,1}$) to a positive shock to housing production ($\Delta B_1 = \Delta B_2 > 0$)



Panel B: Response of savings (ΔW_1) to a positive shock to goods production ($\Delta A_1 > 0$)



Panel D: Response of savings (ΔW_1) to a positive shock to housing production ($\Delta B_1 > 0$)

Figure 2.3 presents selected comparative statics of the discount rate and savings. Panels A and B show the response of the discount rate and savings, respectively, to a positive shock to the goods production for different values of ρ and θ. Panels C and D show the response of the discount rate and savings, respectively, to a positive shock to housing production for different values of ρ and θ.

Table 2.1: Effects on the discount factor.

	$\Delta\phi_{0,1}$	$\Delta\phi_{1,2}$
$\Delta A_1 > 0$	$-$	$+$
$\Delta A_2 > 0$	≈ 0	$-$
$\Delta A_1 = \Delta A_2 > 0$	$-$	$-$
$\Delta B_1 > 0$	$+$ if ρ is small $-$ if ρ is large	$+$ if θ is small $-$ if θ is large
$\Delta B_2 > 0$	$+$ if θ is large $-$ if θ is small	$+$ if ρ is small $-$ if ρ is large
$\Delta B_1 = \Delta B_2 > 0$	$+$ if ρ is small $-$ if ρ is large	$+$ if ρ is small $-$ if ρ is large

Table 1 presents general-equilibrium effects of different types of technology shock on the discount factor. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at t to the production of good and housing, respectively. $\Delta\phi_{0,1}$ and $\Delta\phi_{1,2}$ refer to the response of the discount factor for the first and the second period, respectively. ρ and θ are the parameters for intra- and inter-temporal substitution, respectively.

2.5.3 Implications for the Risk of Housing

The risk of housing is inferred in a unique way by examining the covariation of the discount factor ($\phi_{1,2}$) and rent growth ($g_{p,2}$).[29] Housing, unlike other assets, has a direct effect on the discount factor via rent growth, because of its dual role as a consumption good and as an asset. A negative covariation between the discount factor and rent growth indicates a positive risk premium, since housing asset generates less cashflow exactly when the household wants more wealth.

The rent responses are governed by the simple principle that a good produced by an efficient technology is relatively cheap. A positive shock to goods production in period t makes composite goods cheap in the period and makes p_t high. ($\Delta A_t > 0 \Rightarrow \Delta p_t > 0$) The opposite is true for a housing shock. ($\Delta B_t > 0 \Rightarrow \Delta p_t < 0$) Rent growth increases with $\Delta A_2 > 0$ and $\Delta B_1 > 0$ while decreasing with $\Delta A_1 > 0$ and $\Delta B_2 > 0$. For $\Delta A_1 = \Delta A_2 > 0$ and $\Delta B_1 = \Delta B_2 > 0$, the effects on rent growth are ambiguous since the rents in both periods change in the same direction.

Table 2.2 summarizes the covariation of the discount factor and rent growth. For $\Delta A_1, \Delta A_2$, and $\Delta B_1 = \Delta B_2$, the covariation is uniformly negative. For $\Delta A_1 = \Delta A_2$, the covariation depends on the sign of rent growth and is generally positive except when ρ and μ are extremely small. For ΔB_2 and ΔB_2, the covariation depends on the sign of discount factor and thus on θ and ρ. In general housing assets are riskier when ρ is smaller and μ is smaller. (Figure 2.4) When the two goods are more complementary and land supply is less elastic, the rent response is larger and the negative covariation is of a larger magnitude.

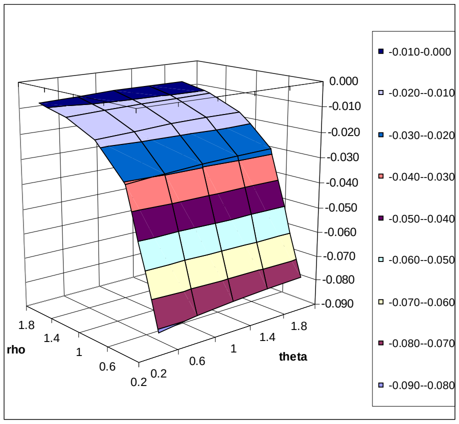
[29]This analysis is distinct from the partial equilibrium analysis in (2.12) since consumption growth is not fixed.

Table 2.2: Covariation of the discount factor and rent growth.

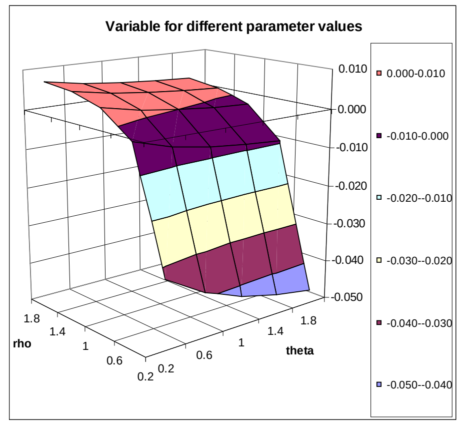
	Covariation
ΔA_1	$-$
ΔA_2	$-$
$\Delta A_1 = \Delta A_2$	$-$ if ρ and μ are small $+$ otherwise
ΔB_1	$+$ if θ is small $-$ if θ is large
ΔB_2	$-$ if ρ is small $+$ if ρ is large
$\Delta B_1 = \Delta B_2$	$-$

Table 2 presents covariation of the discount factor ($\phi_{1,2}$) and rent growth (p_2/p_1) in general equilibrium. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at t to the production of good and housing, respectively. μ is elasticity of land supply. ρ and θ are parameters for intra- and inter-temporal substitution, respectively.

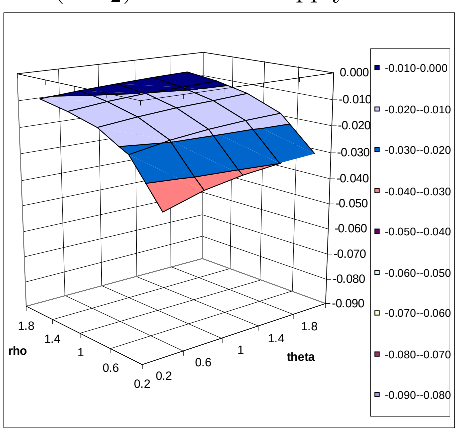
Figure 2.4: Covariation of the discount factor and rent growth.



Panel A: Covariation of the discount factor and rent growth to a shock to goods production (ΔA_2) when land supply is inelastic



Panel C: Covariation of the discount factor and rent growth to a shock to housing production (ΔB_2)



Panel B: Covariation of the discount factor and rent growth to a shock to goods production (ΔA_2) when land supply is elastic

Figure 2.4 presents selected comparative statics of the covariation between the discount factor and rent growth. Covariation is measured in terms of the product of percent changes in the discount factor and rent growth. Panels A and B show the covariation to a shock to goods production for different values of ρ and θ when land supply is inelastic and elastic, respectively. Panel C shows the covariation to a shock to housing production for different values of ρ and θ.

2.5.4 Effects on Asset Prices

Three asset classes are considered: financial assets, housing, and human capital. The prices of housing and human capital are defined as the present discounted values of housing rent and wages, respectively, for a unit amount of the asset:

$$(\text{Housing Price})_0 = \phi_{0,1} p_1 + \phi_{0,1}\phi_{1,2} p_2, \tag{2.11a}$$

$$(\text{Human Capital Price})_0 = \phi_{0,1} w_1 + \phi_{0,1}\phi_{1,2} w_2. \tag{2.11b}$$

The change in the asset price is determined by possibly competing factors on the RHS of (2.11a) and (2.11b).

The financial asset price is equivalent to the price of the installed business capital. Since the price of business capital is always one in the current model (i.e. without a stock adjustment cost), the price with adjustment cost is inferred as follows. If adjustment costs are introduced, the financial asset price would change with the equilibrium level of capital used in goods production (K_t). The price of capital changes since quantity cannot immediately reach the equilibrium level. The price gradually approaches to one as capital is adjusted toward the equilibrium. Therefore, we can regard the change in equilibrium capital as a proxy for the change in capital price.

Effects on Housing Prices

The equilibrium housing price goes up in the following cases.

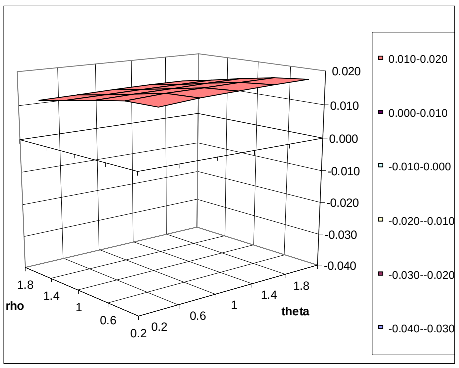
$$
\begin{array}{ll}
\text{Case 1} & \left\{ \begin{array}{l} \text{A positive shock to goods production } (\Delta A_t > 0)\text{, and} \\ \text{inelastic land supply (small } \mu\text{).} \end{array} \right. \\
\text{Case 2} & \left\{ \begin{array}{l} \text{A negative shock to goods production } (\Delta A_t < 0)\text{, and} \\ \text{elastic land supply (large } \mu\text{).} \\ \text{For } \Delta A_2 < 0\text{; additionally, small } \rho \text{ and small } \theta. \end{array} \right. \\
\text{Case 3} & \left\{ \begin{array}{l} \text{A negative shock to goods production in the foreign city.} \\ \text{For } \Delta A_2^* < 0\text{; additionally, elastic land supply (large } \mu\text{),} \\ \qquad\qquad\qquad\qquad \text{small } \rho \text{ and small } \theta. \end{array} \right. \\
\text{Case 4} & \left\{ \text{A negative shock to housing production } (\Delta B_t < 0). \right.
\end{array}
$$

In case 1, the housing rent (p_t) rises at the time of a shock since the numeraire good becomes cheaper. The rent increase is greater if the land supply is more constrained (small μ), since the shift in housing demand results in a greater price change.[30] Although the discount factor ($\phi_{t-1,t}$) and rent may be lower in the other period, the overall effect on housing prices is positive because of a large positive response of rent. With the elasticity of land supply around 0.8 or less, a positive shock leads to the appreciation of housing prices. (Figure 2.5, Panel A) If land supply is more elastic, housing prices exhibit the opposite response, which constitutes Case 2. (Panels A and B) A negative shock to housing production also results in the appreciation of housing prices by increasing rent. (Panel D)

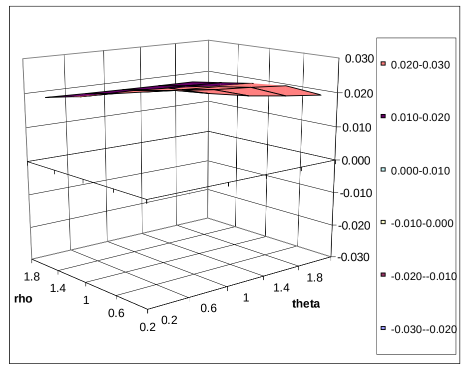
Cases 2 and 3, in which a negative shock to goods production leads to housing price appreciation, provide an interesting insight into the appreciation of housing prices in the United States after 2000. This appreciation occurred in a stagnant economy and with stock prices at a low. A key driver in the model is high future rents induced by reduced housing supply in the future.

[30] The intra-temporal substitution (ρ) also has a secondary effect. If the intra-temporal substitution is low, the price elasticity of housing demand is also low and the rent is more responsive to a shift in supply.

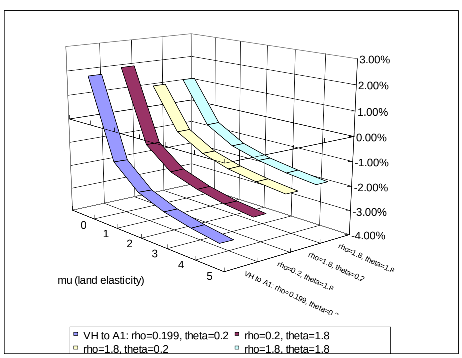
Figure 2.5: Effects of shocks on housing prices



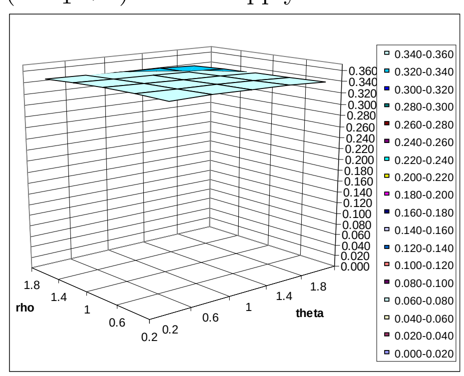
Panel A: (Case 1) Response of housing prices to a positive shock to goods production ($\Delta A_1 > 0$) if land supply is inelastic



Panel C: (Case 2) Response of housing prices to a negative shock to goods production ($\Delta A_1 < 0$) if land supply is elastic



Panel B: (Case 1 and 2) Response of housing prices to a positive shock to goods production ($\Delta A_1 > 0$) for different elasticities of land supply



Panel D: (Case 4) Response of housing prices to a negative shock to housing production ($\Delta B_1 < 0$) if land supply is inelastic

Figure 2.5 presents selected comparative statics of the response of housing prices. Panels A and B show the response of housing prices to a positive shock to goods production against ρ and θ when $\mu = 0$, and against μ, respectively. Panel C shows the response of housing prices to a negative shock to goods production against ρ and θ when $\mu = 5$. Panel D shows the response of housing prices to a negative shock to housing production against ρ and θ when $\mu = 0$.

Consider a current negative shock to goods production of the home city ($\Delta A_1 < 0$) in a land-elastic economy (Case 2). There are competing forces in the housing-price equation (2.11a):

$$\text{(Housing Price)}_0 \;=\; \underset{(+)}{\phi_{0,1}} \; \underset{(-)}{p_1} \;+\; \underset{(+)}{\phi_{0,1}} \; \underset{(-)}{\phi_{1,2}} \; \underset{(+)}{p_2}.$$

The shock lowers the MPK and raises the discount factor (high $\phi_{0,1}$), which helps raise the housing price. The negative shock makes the numeraire good more precious and reduces the current housing rent (low p_1), but the rent reduction is relatively moderate in a supply-elastic city (large μ). The households cash out part of their savings (W_1) in order to support their period 1 consumption (consumption smoothing motive) so that the capital supply at $t = 2$ is reduced. The reduced capital supply results in a higher interest rate or a lower discount factor at $t = 2$ (low $\phi_{1,2}$). When land supply is elastic, the housing rent is more affected by the negative supply shift than the demand shift, which leads to a rise in rent (high p_2). When a higher $\phi_{0,1}$ and p_2 surpass the other competing forces, the housing price appreciates.

In Case 2, we should observe 1) a bull-steepening of the term structure of interest rates (a lower rate at the short end of yield curve), 2) higher expected rent growth, 3) a lower current capitalization rate, or "cap rate", for housing, and 4) reduced savings (attributable to a cashing out of the investment portfolio).[31] Case 2 is also consistent with the negative covariation of the housing price and the interest rate noted by Cocco [2000] and positive covariation of business investment and housing investment noted by Davis and Heathcote

[31] All of these responses were actually observed during the process of housing price appreciation after 2000.

[2005]. While standard two-sector models generate a negative covariation of investments due to the sectoral substitution of capital, the model generates a positive relationship by dint of the capital allocation across cities. A positive covariation between investments, however, means stagnation in near term construction activity after 2000, which is slightly counterfactual.

Improved results are obtained by combining an anticipated negative shock to housing production ($\Delta B_2 < 0$, Case 4) with Case 2. The negative effect of $\Delta A_1 < 0$ on housing structures is mitigated or may even be reversed. All other effects are enhanced: higher housing prices, lower financial asset prices, a steeper slope of yield curve, a higher rent growth, a lower cap rate, and lower savings. This combined case is also appealing because of a better match to a cross-regional observation that housing price appreciation is pronounced in areas with rich housing amenities such as San Diego and Miami. Housing price appreciation seems to be partly driven by a local shock to preference for housing, which is equivalent to a shock to housing production in the model.

Table 2.3: Predictions in four cases of housing price appreciation.

Case 1: A positive shock to good production with inelastic land supply

	$\Delta A_1 > 0$	$\Delta A_1 = \Delta A_2 > 0$	$\Delta A_2 > 0$
Term structure	bear-flattening	bear-parallel shift	bear-steepening
Future rent growth	$-$	$-$	$+$
Current cap rate	$+$	$+$	$+$ if θ small $-$ if θ large
Savings	$+$	$+$	$-$
$\text{Cov}(K_1, S_1)$	$+$	$+$	≈ 0

Case 2: A negative shock to good production with elastic land supply

	$\Delta A_1 < 0$	$\Delta A_1 = \Delta A_2 < 0$	$\Delta A_2 < 0$
Term structure	bull-steepening	bull-parallel shift	bull-flattening
Future rent growth	$+$	$+$	$-$
Current cap rate	$-$	$-$	$+$ if θ large $-$ if θ small
Savings	$-$	$-$	$+$
$\text{Cov}(K_1, S_1)$	$+$	$+$	≈ 0

Case 3: A negative shock to foreign city

	$\Delta A_1^* < 0$	$\Delta A_1^* = \Delta A_2^* < 0$	$\Delta A_2^* < 0$
Term structure	bull-steepening	bull-parallel shift	bull-flattening
Future rent growth	$+$	$+$	$-$
Current cap rate	$-$	$-$	$+$ if θ large $-$ if θ small
Savings	$-$	$-$	$-$ if $\theta \& \rho$ small $+$ otherwise
$\text{Cov}(K_1, S_1)$	$+$	$+$	≈ 0

Case 4: A negative shock to housing production

	$\Delta B_1 < 0$	$\Delta B_1 = \Delta B_2 < 0$	$\Delta B_2 < 0$
Term structure	mixed	mixed	mixed
Future rent growth	$-$	$+$ if $\rho > 1$ $-$ if $\rho < 1$	$+$
Current cap rate	$+$	$+$ if $\rho > 1$ $-$ if $\rho < 1$	$-$
Savings	$+$ if θ small $-$ if θ large	$+$ if $\rho > 1$ $-$ if $\rho < 1$	$+$ if $\theta > 1$ $-$ if $\theta < 1$
$\text{Cov}(K_1, S_1)$	$-$	$-$	$-$

Table 2.3 presents model predictions in the four cases of housing price appreciation. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at t to the production of good and housing, respectively. "Mixed" response refers to more complex comparative statics.

Table 2.3 summarizes the model predictions for all four cases. Either Case 2 with $\Delta A_1 < 0$ (elastic land supply) or Case 3 with $\Delta A_1^* < 0$ (a negative shock in the foreign city) provides the predictions that fit best the situation after 2000. Case 3 is driven by the capital flow from the home city under recession. Case 4 is mainly driven by a higher rent due to less efficient housing production. In this case the covariation of investments is negative due to capital substitution between sectors.

Effects on Human Capital and Financial Assets

Table 2.4 presents the effects of various technology shocks on asset prices. The value of human capital rises with a positive shock to goods production ($\Delta A_t > 0$) mainly because of a large increase in wages (w_t) with an inelastic labor supply. (Note the second column of Table 4) A positive shock to housing production ($\Delta B_t > 0$) generates parameter-dependent effects. When the two goods are complementary (small ρ), the value of human capital rises because greater demand for composite goods (Y_t) increases wages. Inter-temporal substitution (θ) also affects the value via variations in the discount factor that are discussed in Section 5.2.

The price of the financial asset exhibits very similar responses as the value of human capital. The price rises with a positive shock to goods production. (The third column of Table 4) A positive shock at $t = 1$ ($\Delta A_1 > 0$ or $\Delta A_1 = \Delta A_2 > 0$), for example, will raise the price of the financial asset since the equilibrium levels of K_1 and K_2 are higher. A higher productivity leads to more capital, either due to the substitution for housing production in the same city or the substitution for foreign production. A positive shock to housing production also generates the parameter-dependent effects that are very similar to the case

Table 2.4: Effects of technology shocks on asset prices.

<table>
<tr><th></th><th>Housing Asset</th><th>Human Capital</th><th>Financial Assets</th></tr>
<tr><td>$\Delta A_1 > 0$</td><td>$+$ if μ small
$-$ if μ large</td><td>$+$</td><td>$+$</td></tr>
<tr><td>$\Delta A_2 > 0$</td><td>$-$ if $\begin{cases} \mu \text{ large} \\ \rho \text{ small} \\ \theta \text{ small} \end{cases}$
$+$ otherwise</td><td>$+$</td><td>$+$ if θ large
$-$ if θ small</td></tr>
<tr><td>$\Delta A_1 =$
$\Delta A_2 > 0$</td><td>$+$ if μ small
$-$ if μ large</td><td>$+$</td><td>$+$</td></tr>
<tr><td>$\Delta B_1 > 0$</td><td>$-$</td><td>$-$ if $\begin{cases} \rho \text{ large} \\ \theta \text{ large} \end{cases}$
$+$ otherwise</td><td>$-$ if $\begin{cases} \rho \text{ large} \\ \theta \text{ large} \end{cases}$
$+$ otherwise</td></tr>
<tr><td>$\Delta B_2 > 0$</td><td>$-$</td><td>$-$ if $\begin{cases} \rho \text{ large} \\ \theta \text{ small} \end{cases}$
$+$ otherwise</td><td>$+$ if θ large
$-$ if θ small</td></tr>
<tr><td>$\Delta B_1 =$
$\Delta B_2 > 0$</td><td>$-$</td><td>$+$ if $\rho < 1$
$-$ if $\rho > 1$</td><td>$+$ if $\rho < 1$
$-$ if $\rho > 1$</td></tr>
</table>

Table 2.4 presents effects of different types of technology shock on asset prices. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at t to the production of good and housing, respectively. μ is elasticity of land supply. ρ and θ are parameters for intra- and inter-temporal substitution, respectively.

of human capital.

2.5.5 Covariation of Asset Prices

Now we examine the covariation of different asset prices. The covariation in response to a shock is measured in terms of the product of the percentage changes in the two prices.

Financial Assets and Human Capital

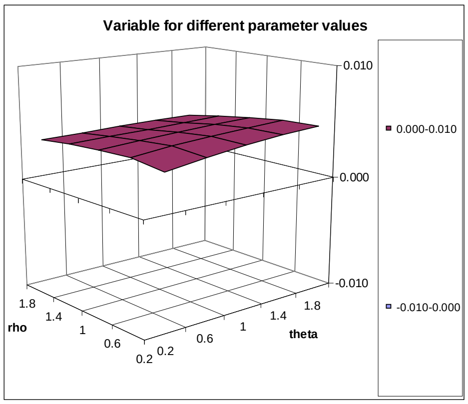
As seen in Table 2.4, most of the time the price of financial assets and the value of human capital move in the same direction. This is because a change in productivity affects both capital demand and labor demand in the same way when a shock is given at $t = 1$ (ΔA_1 and $\Delta A_1 = \Delta A_2$). When a shock is anticipated in the future (ΔA_2 and ΔB_2), they may move in opposite directions. For example, given a positive shock to goods production in period 2 ($\Delta A_2 > 0$), the household also wants to consume more at $t = 1$ if the inter-temporal substitution is low (small θ). However, housing services must be produced locally while composite goods can be imported from the foreign city. Therefore, capital at $t = 1$ is allocated more to housing production and the amount of capital dedicated to goods production (K_1) is reduced. Therefore, prices of financial assets and human capital may move in opposite directions when inter-temporal substitution is low.

Housing and Other Assets

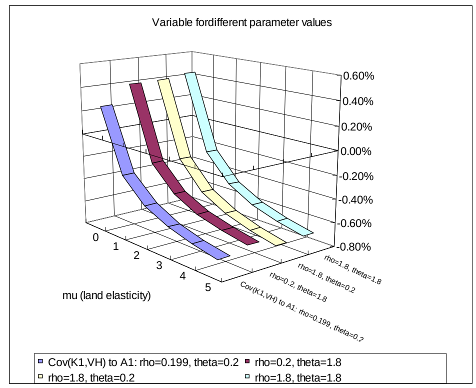
The covariation of housing price and the value of human capital depends on the supply elasticity of land (μ) and the elasticities in the utility function (ρ and θ). The effect of a shock to goods production (ΔA_t) on this covariation is determined by the sign of the change in housing prices since the response of human capital is uniform. For example, in response to a positive shock, the human capital always appreciates due to wage increases. As seen in Figure 2.6, Panel A housing prices and human capital vary together when an inelastic land supply (small μ) makes the housing rent more responsive to a positive demand shock. Conversely, the covariation is negative when relatively elastic land supply (large μ) makes the rent more stable. (Panel B) The critical value of μ is different for different types of shocks but is not so large for ΔA_1 (Panel C) and $\Delta A_1 = \Delta A_2$. ($\mu \cong 0.8$ for $\Delta A_1 > 0$ and $\mu \cong 2$ for $\Delta A_1 = \Delta A_2 > 0$)

With a shock to housing production (ΔB_t), the link between housing prices and human capital is determined by the effect on human capital. Housing prices always depreciate with a positive shock and appreciate with a negative shock, regardless of parameters. The covariation of housing prices and human capital is generally negative when the two goods are more complementary (small ρ) and when the inter-temporal substitution is low (small θ). (Panel D) With a positive shock, for example, human capital appreciates if the two goods are complementary. This is because reduced housing expenditures lead to a lower interest rate, which stimulates production of composite goods.

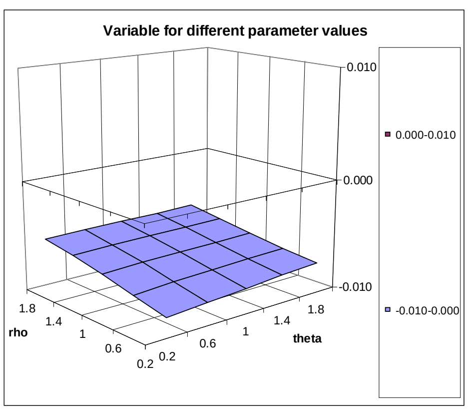
Figure 2.6: Covariation of asset prices.



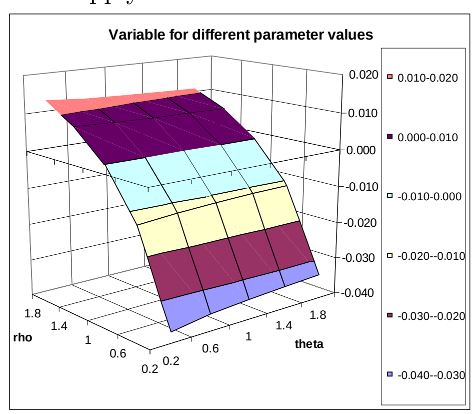
Panel A: Covariation of housing prices and human capital in response to a shock to goods production (ΔA_1) if land supply is inelastic



Panel C: Covariation of housing prices and human capital in response to a shock to goods production (ΔA_1) for different elasticities of land supply



Panel B: Covariation of housing prices and human capital in response to a shock to goods production (ΔA_1) if land supply is elastic



Panel D: Covariation of housing prices and human capital in response to a shock to housing production ($\Delta B_1 = \Delta B_2$) if land supply is inelastic

Figure 2.6 presents covariation of home prices and human capital, which is measured in terms of the product of percent changes in home prices and human capital. Panels A and B show the covariation in response to a shock to goods production against ρ and θ when $\mu = 0$ and $\mu = 5$, respectively. Panel C shows the covariation in response to a shock to goods production against μ. Panel D shows covariation in response to a shock to housing production against ρ and θ when $\mu = 0$.

The covariation between the prices of housing and the financial asset is similar to that between the housing price and the human capital. This is because of the general comovement of human capital and financial assets.

Proposition 5 *Housing assets are a hedge against human capital risk and the financial risk if*

$$\textit{1)}\begin{cases}\textit{the land supply is sufficiently elastic (large } \mu\textit{)} \\ \textit{when the source of risk is a current shock to goods production, or}\end{cases}$$
$$\textit{2)}\begin{cases}\textit{the two goods are more complementary (small } \rho\textit{)} \\ \textit{when the source of risk is a shock to housing production.}\end{cases}$$

A positive production shock causes declines in both the housing price and the value of human capital in the foreign city due to a lower discount factor and diminished production of both goods. A housing production shock has a very small impact on the foreign city, so that the covariation is close to zero.

Cross-Country Differences in Asset Price Covariation

A stylized fact, in the US, is that the correlation between the housing prices and stock prices is negative (or at least close to zero). These empirical findings suggest that housing assets provide at least a good diversification benefit and may even be a hedge against the financial risk.[32] An illustrative sample period is after 2000, during which stock prices were depressed and housing prices appreciated. In Japan, the correlation is much

[32] Cocco [2000] and Flavin and Yamashita [2002] among others note the negative correlation. Goetzmann and Spiegel [2000] find a negative Sharpe ratio for housing, which is consistent with the opportunity for hedging.

higher.[33] Illustrative periods are the 1980's and the 90's. In the 80's both stock prices and housing prices appreciated, but in the 90's both were depressed. The relationships between housing and human capital, and between human capital and stock are probably positive in both countries although the results are mixed.[34]

This chapter provides a rational foundation to explain this difference between countries in the covariation structure among the three assets. A standard explanation for a positive covariation of housing prices and stock prices in Japan relies on monetary policy. It treats both stocks and real estate the same, focusing on the nominal values of these assets. It does not explain why we observe negative covariation in the U.S. Another explanation is more "behavioral." Japanese households and investors are somehow more prone to irrational exuberance and an investment boom spreads across assets. This chapter's explanation, based on the land supply elasticity, is more natural and matches a key difference between the two economies.

It is important to understand properly the land supply in our model. The land supply is obviously most restricted by the topographic conditions and population densities. Table 5 compares the per capita habitable areas for five countries. The habitable area is the gross usable area, excluding forests and lakes. The U.S. has 25 times more habitable area per capita than Japan. The ability to supply housing, whether by land development or via infill, has been much more limited in Japan. Other supply constraints are imposed by the regulatory system and the adjustment speed of housing stock. European countries

[33] Quan and Titman [1999] reports a high correlation in Japan between stock and commercial real estate, which is positively correlated with housing prices. Casual observation after 1970 also confirms this.

[34] Cocco [2000] reports a positive correlation between housing and labor income. Davidoff [2006] also obtains a positive point estimate but it is not significantly different from zero. The correlation between stock and wage income is low in the short term but the correlation is much higher for proprietary business income (Heaton and Lucas [2000]) and in the long run (Benzoni et al. [2005]).

Table 2.5: Per Capita Habitable Area.

	Population (A) $(1,000s)$	Habitable Area (B) $(1,000km^2)$	(B) / (A) $(1,000m^2/person)$
U.S.	$288,369$	$7,193$	24.9
France	$59,486$	401	6.7
Italy	$57,482$	236	4.1
Germany	$82,488$	249	3.0
Japan	$127,480$	122	1.0

Table 2.5 presents population, habitable area, and per capita habitable area for five countries. Population is an estimate for 2002 by the United Nations (Demographic Yearbook 2002). The habitable area is the total area less forests and lakes in 2001.

such as Germany generally impose stricter environmental and historical restrictions on new developments. Such restrictions make the land supply more inelastic than the level implied by topographic conditions. The adjustment speed of housing stock is also affected by negotiation practices. Many Japanese redevelopment projects take more than twenty years to complete due to the prolonged negotiation process. Such slow adjustment functions as a short-run inelasticity of supply.[35]

[35]Quigley and Raphael [2005] and Green et al. [2005] find that population density and housing-market regulation limit housing supply in the U.S. Edelstein and Paul [2000] discuss factors that severely limit land supply in Japan.

Implications for stock-market participation and homeownership

Positive covariations among three broad asset classes have important implications for the stock-market participation and homeownership. With positive covariations, the optimal portfolio choice results in a small position (or even a short position) in the asset that can be adjusted more freely.[36] In general, there are few constraints on financial asset levels, while human capital and homeownership are constrained at some positive levels. Under these constraints, positive covariations in prices lead to less holdings of financial assets, or limited stock-market participation, as derived by Benzoni et al. [2005]. They note an empirical fact that human capital and stock prices are more highly correlated in the long run, and they show that, assuming co-integrated prices of these two assets, the optimal portfolio may be even to short-sell stocks, especially for younger investors. If the rental housing market is well functioning and households are relatively free to choose their level of housing asset holdings, positive covariations lead to less homeownership. This is examined by Davidoff [2006], who shows that households with a higher correlation between labor income and housing prices own less housing.

The current model derives a positive covariation between human capital and financial assets, rather than just assuming one, for most cases, and between housing and financial assets depending on the parameters. Thus, the model provides a rationale for the limited-market participation. More interestingly, the model predicts more severe limitations in stock-market participation in a land-inelastic economy, since all three assets (human capital, housing, and stock) are positively related in such an economy. In a land-elastic economy,

[36] The partial equilibrium portfolio choice literature leads to the conclusion that less holding of stock is optimal if the exogenously given covariance is positive and vice versa. See Flavin and Yamashita [2002], Cocco [2004], and Cauley et al. [2005].

housing assets serve as a hedge against the other assets and the limitation is less severe. Market participation is still limited due to the positive covariation between financial assets and human capital (the largest component of asset holdings).[37]

This provides a plausible explanation for the fact that Japanese investors participate less in the stock market than the U.S. investors. The Japanese rental housing markets have not functioned well due to the tenancy law that heavily protects tenants' rights. The government also favors homeownership through subsidized financing and tax treatments of housing.[38] Households are economically encouraged to hold large housing assets, despite the positive correlations with other assets. Therefore, the optimal portfolio includes a less stock. Again, previous explanations tend to rely on the "irrationality", differences in "culture", or differences in skills and experience. In fact, based on such arguments, the Japanese government has adopted policies to encourage stock market participation, measures supported by the financial industry. Our result provides a counter argument: namely, that less participation in the stock market is a perfectly rational choice for households in the land-inelastic Japanese economy.

2.6 Conclusion and Discussion of Uncertainty

In this chapter, I build a simple deterministic model of a production economy to study the general equilibrium relationship between the business cycles and asset prices, with an emphasis on implications to portfolio choice.

The first of my two main results is that the supply elasticity of land plays a

[37] Cocco [2004], using PSID, estimates that the 60-85% of the total assets are human capital.
[38] Kanemoto [1997] discusses in detail homeownership and limited rental markets in Japan.

significant role in determining the covariations of asset prices. In particular, a negative productivity shock to goods production may lead to a housing price appreciation if land supply is elastic. A key driver is a higher housing rent in the future resulting from a reduced housing supply. This implies that an economy with an inelastic housing supply is more likely to exhibit a positive price covariation between housing and other assets and thus, either less stock-market participation or less homeownership. These predictions are broadly consistent with the differences between the U.S. and Japan.

The second result is that growth of housing rent alters the marginal utility of consumption when the utility function is non-separable in housing and other goods. For example, the marginal utility will be adjusted upward (implying that consumers are less satisfied) if rent growth is lower, provided that the two goods are complementary. The rent growth factor may mitigate the well-known puzzles on the equity premium and the risk-free rate, either by magnifying consumption variation or imposing a downward bias on the estimate of the IES. The risk of housing assets is also inferred from the characterization.

This chapter suggests a rich array of opportunities for empirical analysis. First, the new characterization of the discount factor allows us to use housing rent data to estimate elasticities of substitution. Housing rent data have several advantages over housing consumption data: 1) rent data can be more accurately collected, 2) rents respond more sharply to changes in economic conditions, and 3) more detailed regional data are available for rents. Second, cross-country variations in land supply elasticity and elasticities of substitution can be used to test (or estimated jointly with) the predictions of the model on 1) the covariance between housing prices and other asset prices, 2) the level of stock holdings,

3) effects of technology shocks on housing prices, interest rates, cap rate, and savings, 4) volatility of the discount factor, and 5) risk premium of housing assets. The model also predicts cross-city variations in consumption, investment, and rent as well as cross-sector variations in production and investment although these results are not presented in detail for brevity.

The next task of this research will be to accommodate uncertainty explicitly. In a dynamic stochastic setting, the risk arising from price dynamics also affects the asset allocation, as risk-adjusted returns are equated in equilibrium. The covariance structure between asset returns and the discount factor, and thus, the risk premium with respect to different types of business cycles can be explicitly shown. The system may be solved either by second-order approximation or by numerical methods. Although the method often used in the literature is linear approximation, the certainty equivalence property resulting is not suitable for the study of asset prices. By calibration, the levels of variables can be discussed rather than just the direction and the relative magnitudes as done in the present paper. Other directions of future extension include modeling imperfect household mobility across cities and allowing for non-homothetic preferences. Household mobility will enrich the results by making the labor supply effectively more elastic and generating predictions about city size. Non-homothetic preferences will also be important when the model is calibrated to the data. There is evidence that the parameter estimates will be quite different from the CES-case. Since the relative elasticities of substitution are critical to the dynamics, this issue needs to be examined with particular care.

Chapter 3

Estimating Elasticities of Substitution for Non-Homothetic Preferences over Housing and Non-Housing

3.1 Introduction

Consumption substitution between different goods within a period and substitution between different periods are important for asset pricing. It is because risks of asset returns are ultimately measured by covariance with marginal utility of consumption, which depends on both static substitution and intertemporal substitution. Early empirical studies focused on the intertemporal elasticity of substitution (IES) based on single-good models, by implicitly assuming separability of utility function. More recently, it is shown that both IES and the static elasticity of substitution (SES) have important implications on asset pricing.[1]

In Chapte2, I build a two-good equilibrium model with a non-separable utility function, and shows that the asset pricing kernel for time t and $t+1$, $\phi_{t,t+1}$, is expressed as

$$\phi_{t,t+1} = \beta \left[g^c_{t,t+1} \times {g^p_{t,t+1}}^{\theta-\rho} \right]^{-\frac{1}{\theta}},$$

where β is subjective discount factor, $g^c_{t,t+1}$ is consumption growth of the first good (numeraire good), $g^p_{t,t+1}$ is growth rate of a price index of the second good, θ is IES, and ρ is SES. Dynamic equilibrium relationship among $\phi_{t,t+1}, g^c_{t,t+1}$,and$g^p_{t,t+1}$ is determined by $\theta-\rho$, or a measure of relative complementarity between two goods.

Intuitively, given a change in relative price of the second good, consumers change their consumption basket within the period and try to smooth consumption over time if ρ is relatively high ($\theta-\rho<0$). If, on the contrary, θ is relatively high ($\theta-\rho>0$), they prefer to change their consumption pattern over time while keeping consumption basket within the period relatively fixed. As a result, $g^p_{t,t+1}$ may or may not magnify consumption growth depending on the sign of $\theta-\rho$. It is shown that relative complementarity (i.e. $\theta-\rho>0$) is generally needed to match models with observed facts.

Empirical estimation of IES and SES is usually done for durable and non-durable goods.[2] However, housing has started to draw more attention than ever, given its signif-

[1] A seminal paper on single-good asset pricing is Hansen and Singleton [1983]. Multiple-good models are explored by Piazzesi et al. [2004], Lustig and van Nieuwerburgh [2004b], Pakos [2003], Yogo [2005] among others.

[2] The estimates of IES are 0.5˜2.0 (Hansen and Singleton [1983]), 0.4 (Ogaki and Reinhart [1998]), 0.02 (Yogo [2005]), 1˜2 (Vissing-Jorgensen and Attanasio [2003] and Bansal and Yaron [2004]). The estimates of SES are 1.17 (Ogaki and Reinhart [1998]), 0.52˜0.79 (Yogo [2005]). Between traded and non-traded goods, Tesar [1993] finds SES of 0.44.

icance in consumption basket and its role as a major asset class. While there are several theoretical studies (e.g. Piazzesi et al. [2004], Lustig and van Nieuwerburgh [2004b], and Yoshida [2006]), very little has been done in estimating IES and SES for housing and non-housing goods.[3]

A notable exception is Davis and Martin [2005]. They obtain high SES between housing and non-housing of 2.2 or higher, by applying a CES utility function to aggregate time-series data. The CES utility function, which nests the Cobb-Douglas and log-linear utility functions, is a workhorse in empirical research. A high estimate of SES, however, is not consistent with anecdotal evidence or results from previous literature on housing demand. A high SES is associated with a high price elasticity of housing demand, implying that households spend less on housing if housing price is high. This prediction is opposite from casual observation that households generally put a higher expenditure weight on housing in cities with high housing costs. A large body of literature on housing demand also reaches a consensus that price elasticity of housing demand is significantly less than one.[4]

Our conjecture is that homotheticity of preferences, implicitly built in the CES function, is a cause of the high estimate of SES. Homotheticity of preferences is a property that income expansion paths of consumption choice are rays from the origin; in other words, consumption share is invariant to income levels, ceteris paribus. If, however, income elasticities are different across goods, preferences exhibit non-homothetic property. By imposing homotheticity restriction in estimating SES, any variation in consumption share must be attributed to a price change, even if actual variation in consumption share comes

[3] In calibrating models, Lustig and van Nieuwerburgh [2004b] set $\rho \in [0.05, 0.75]$ and Piazzesi et al. [2004] set $\rho \in [1.01, 1.10]$

[4] Rough consensus on the price elasticity is between -0.8 and -0.3. See, for example, literature reviews by Mayo [1981], Harmon [1988], Ermisch et al. [1996] and Green and Malpezzi [2003].

from income growth.

In this chapter, we estimate SES between housing and non-housing goods together with IES using aggregate time-series data, allowing for non-homothetic preferences. Specifically, we adopt the generalized elasticity of substitution (GES) form proposed by Pakos [2003]. Further, we examine properties of non-homothetic preferences by estimating the effects of income level and income composition on consumption share. Our estimations reject homotheticity and give low SES between 0.4 and 0.9 when we allow for non-homotheticity. We find an upward bias in estimates of SES under the homotheticity assumption. Estimates of IES are quite low between 0.05 and 0.14, which are consistent with early studies. Although our plain estimates indicate $\theta - \rho < 0$, eliminating downward biases in estimates of IES would reverse the sign.

We also show that consumption share of housing decreases as income grows, and that it increases as more income is derived from investments. The results imply that housing demand is less elastic to income changes than non-housing, and that investors have a smaller SES than employees. Finally, we confirm the same result based on the Stone-Geary utility function. We obtain additional evidence of non-homotheticity by finding that subsistence levels of both consumption goods are significantly positive. We also confirm that consumption share of housing decreases as income grows.

The organization of the paper is as follows. In Section 2, we estimate SES, IES and other parameters of consumers' utility function by GMM, using moment conditions derived as the first-order conditions of consumers' problem. In Section 3, we estimate SES by linear regressions of consumption ratio of housing on prices. We also test if homotheticity of preferences is rejected by the data. In Section 4, we report regression results of linear expenditure equation based on the Stone-Geary utility function. Section 5 concludes.

3.2 Estimating SES and IES with the GES utility function

3.2.1 Model

Following Pakos [2003], we use the generalized elasticity of substitution (GES) utility function, which allows for non-homothetic property. The function is a modified CES function with an additional parameter η:

$$u\left(C,H\right)=\left[\left(1-\alpha\right)C^{1-1/\rho}+\alpha H^{1-\eta/\rho}\right]^{1/(1-1/\rho)}, \tag{3.0}$$

where C_t is the consumption of non-housing good and H_t is the consumption of housing services at time t. $\alpha\in(0,1)$, and $\rho\geq 0$ is SES between two goods.

It nests the CES utility function as a special case of $\eta=1$. The CES utility function, which is a workhorse in the asset pricing research, exhibits homothetic property so that any change in consumption ratio must be induced by a price change. The CES utility function itself nests the Cobb-Douglas utility function ($\rho=1$) and the log-linear utility function ($\rho=\theta=1$).

If η is different from one, preferences are non-homothetic. As shown in Appendix 2, $\eta<1$ implies that the consumption ratio of housing services to non-housing rises as income grows: consumption share of housing is positively related to income level.

In this section, we derive the first-order conditions that serve as moment conditions in the estimation procedure. We will present estimation results both for the CES utility case with $\eta = 1$ and for the GES utility function with no restriction on η.

The consumer's dynamic problem is

$$\max_{\{C_t, H_t, \mathbf{q}_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \Omega \left[u \left(C_t, H_t \right) \right] \right]$$

$$s.t.\ \ \forall t: \quad C_t + P_t H_t + S_t = Y_t + S_{t-1} \left(\mathbf{q}'_{t-1} \mathbf{R}_t \right),$$

$$\mathbf{q}'_t \iota = 1,$$

where $E_t[\cdot]$ is the expectation conditional on the information set at time t, β is the consumer's subjective discount factor, $\Omega[X] \equiv X^{(1-1/\theta)} / (1 - 1/\theta)$, θ is the IES, P_t is the relative price of housing services at time t, and S_t is the consumer's asset holdings at time t, Y_t is exogenous labor income at time t, $\mathbf{q}_t$ is N-vector of portfolio weights based on the information set at time t, and $\mathbf{R}_t$ is N-vector of gross return to each asset in the portfolio at time t, ι is N-vector of ones. Therefore, $S_{t-1} \left(\mathbf{q}'_{t-1} \mathbf{R}_t \right)$ is the total return to the consumer's portfolio at time t in units of the numeraire good.

The competitive equilibrium agrees with the optimal allocation of the social planner in this complete-markets economy with the representative consumer. The planner's dynamic programming consists of the following Bellman equation and the accumulation equation.

$$\begin{aligned} \forall t;\ \ V_t \left(S_{t-1} \right) &= \max_{C_t, H_t} \left\{ \Omega \left[u \left(C_t, H_t \right) \right] + \beta E_t \left[V_{t+1} \left(S_t \right) \right] \right\} \\ s.t.\ \ S_t &= Y_t + S_{t-1} \left(\mathbf{q}'_{t-1} \mathbf{R}_t \right) - C_t - P_t H_t, \end{aligned}$$

where $V_t(S_{t-1})$ is the value function at time t given S_{t-1}. From the first-order conditions, we obtain the following intra- and inter-temporal optimality conditions:

$$P_t = \frac{\Omega_{H,t}}{\Omega_{C,t}} = \frac{(\rho-\eta)\,\alpha}{(\rho-1)\,(1-\alpha)}\left(\frac{H_t^{\eta}}{C_t}\right)^{-\frac{1}{\rho}}, \tag{3.-3}$$

$$1 = E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}R_{t+1}^{j}\right], \tag{3.-2}$$

where R_{t+1}^{j} is the gross return to any particular asset j in the portfolio at time $t+1$. The derivation is shown in Appendix B. We are going to use these two conditions as moment conditions in the empirical part. The pricing kernel $\beta\Omega_{C,t+1}/\Omega_{C,t}$ is expressed in terms of consumption ratios of two goods, or equivalently, prices of housing services.

$$\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}} = \beta\left\{\left(\frac{C_{t+1}}{C_t}\right)\left[\frac{(1-\alpha)+\alpha\left(\frac{H_{t+1}^{\eta}}{C_{t+1}}\right)^{1-\frac{1}{\rho}}}{(1-\alpha)+\alpha\left(\frac{H_t^{\eta}}{C_t}\right)^{1-\frac{1}{\rho}}}\right]^{\frac{\theta-\rho}{1-\rho}}\right\}^{-\frac{1}{\theta}} \qquad (PK1)\ (3.-1)$$

$$= \beta\left\{\left(\frac{C_{t+1}}{C_t}\right)\left[\frac{\left[(1-\alpha)^{\rho}+\alpha^{\rho}\left(\frac{\rho-1}{\rho-\eta}P_{t+1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[(1-\alpha)^{\rho}+\alpha^{\rho}\left(\frac{\rho-1}{\rho-\eta}P_t\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}\right]^{\theta-\rho}\right\}^{-\frac{1}{\theta}} \qquad (PK2)\ (3.0)$$

We use both expressions in order to utilize different data sets.

3.2.2 Estimation Strategy

We estimate five parameters $(\rho, \theta, \alpha, \beta, \eta)$ by generalized method of moments (GMM) with moment conditions (3.-3) and (3.-2). By using return data of N assets including risk-free asset and I instruments as conditioning information at time t, we have $I(N+1)$ moment conditions. (IN from inter-temporal condition (3.-2) and I from intra-temporal condition (3.-3)) We use market return and risk-free rate as return data. As instruments, we use constant for unconditional moments, and lagged consumptions (C and H) and returns for conditional moments. By using three moment conditions and five instruments, we have 25 over-identifying restrictions.

We adopt the two-step (efficient) generalized method of moments (GMM), in which the identity matrix is used as the weighting matrix in the first stage and then the inverse of the asymptotic covariance matrix estimated from the first stage is used in the second stage. The second-stage weighting matrix is estimated by the Bartlett kernel of Newey and West (1987) with 6 lags.

3.2.3 Data

We use data for consumption of housing and non-housing, price of non-housing relative to that of non-housing, and asset returns. We use two different data for housing consumption: one is housing services and the other is housing stock. We adopt the beginning-of-period convention, following Campbell (2003) and Yogo (2005) among others. For example, the consumption during 2000:1 is treated as if all the consumption took place at the beginning of 2000:1. Prices and stocks are all measured at the beginning of each period. Use of end-of-period convention does not alter our results.

Relative price of housing consumption: There are two data series available for price of housing services: 1) Personal Consumption Expenditure (PCE) price index for housing taken from National Income and Products Accounts (NIPA) published by the Bureau of Economic Analysis (BEA) and 2) Consumer Price Index (CPI) for shelter published by the Bureau of Labor Statistics (BLS). Both statistics currently use the rental equivalence approach to estimating the shelter costs of homeowners. The owners' equivalent rent is obtained from corresponding market rental value of homes that have the same size, quality and type. In equilibrium of frictionless markets, owners' equivalent rent is equal to the user cost of housing. The rental equivalence approach is less prone to model errors, especially errors on expected future appreciation of housing asset prices, than the user cost approach.

We use NIPA-based PCE price index for housing (Table 2.3.4 - Price Indexes for Personal Consumption Expenditures, Line 14) because CPI for shelter is discontinuous in 1983 due to a major methodological change. Prior to 1983, BLS adopted the "asset price" approach by which the shelter costs consist of five elements: 1) home purchase cost, 2) mortgage interest cost, 3) property taxes, 4) homeowner insurance charges and 5) maintenance and repair costs. The asset price approach is irrelevant especially because housing asset prices for purchase are directly used for price of housing services. The CPI shelter price is driven by large changes in housing asset prices in the 70's and the early 80's.[5]

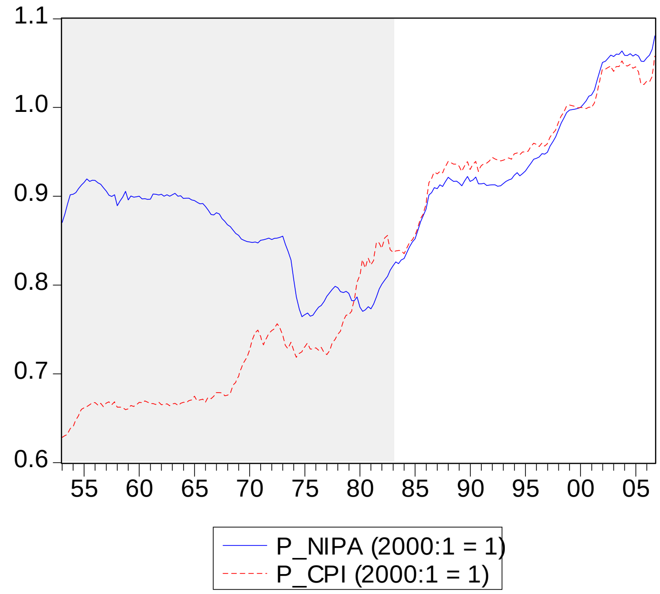
Figure 3.1 plots two data series for price of housing service. Panel A is relative price of housing deflated by non-housing prices and Panel B is the growth rates of price indexes of housing. As apparent in the figure, two series are closely related from 1983, but they exhibit a wide gap prior to 1983 (shaded region).

[5] There are other differences between two indexes. First, CPI is a Laspeyres index (i.e. it is constructed with a fixed basket of items) while NIPA index is a Paasche or "chain-type" index. (i.e. composition of items changes every quarter) CPI weights are only renewed every two years.

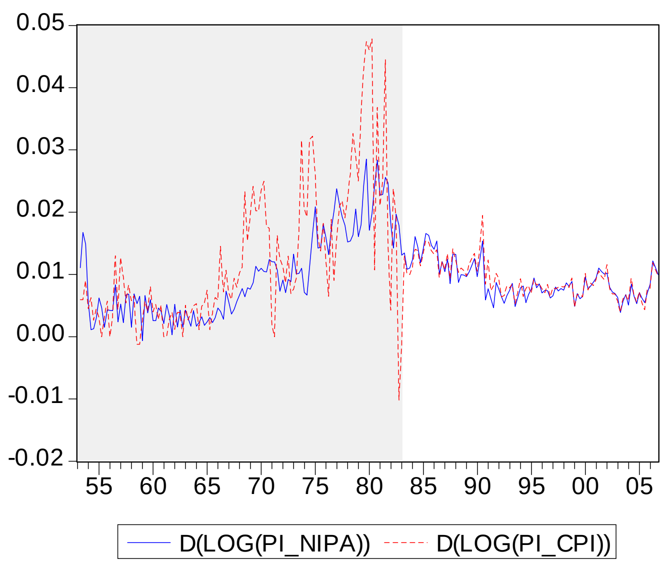
Second, PCE price index and CPI use very different weights on housing. As of December 2004, PCE puts 15% on shelter while CPI puts 32.7%. The gap is partly due to different coverages of items; CPI covers only about three quarters of PCE price index. CPI captures out-of-pocket expenditures made by households while PCE also includes expenditures made on behalf of households (e.g. medical care paid by employers and governments). See Poole et al. [2005], Moyer [2006] and McCully [2006] for detailed discussion.

Figure 3.1: Prices of Housing Services: NIPA-PCE and CPI

Panel A: Price of Housing Relative to Non-Housing



Panel B: Growth Rates of Housing Price Indexes



We deflate NIPA-based PCE price index of housing by that of non-housing. Non-housing includes non-durables, durables, and services but housing. Since non-housing defined in this study is not directly reported in NIPA tables, price index for non-housing is calculated by dividing the nominal expenditure by the real expenditure of non-housing items.

Non-housing consumption: Non-housing consumption data is per capita real consumption expenditure on non-housing. To obtain the data series, first, nominal personal consumption expenditure is taken from NIPA Table 2.3.5. (Personal Consumption Expenditures by Major Type of Product) published by the BEA. Then the real personal consumption expenditure in 2000 dollar is calculated for each product type by using the chain-type price indexes that are taken from NIPA Table 2.3.4. (Price Indexes for Personal Consumption Expenditures by Major Type of Product) We subtract real housing services from real total consumption expenditure. The data is converted to per capita basis by dividing by the population published by the Census Bureau (taken from freelunch.com).

Housing Consumption: We use two different measures of housing consumption. The first is simply per capita real personal consumption expenditure for housing services, which is constructed in the same way as non-housing consumption.

The second is rescaled per capita real housing stock excluding land. The underlying assumption is a linear transformation technology from housing stock to housing services. We constructed quarterly data series for housing stock by using two different methods. The first data for residential stock is the Net Stock of Fixed Reproducible Tangible Wealth for 1925-1997 reported in the SURVEY OF CURRENT BUSINESS, September 1998 by BEA. It provides annual end-of-year estimate of net residential capital. The data is splined up to 2004 by the Detailed Data for Fixed Assets and Consumer Durable Goods from BEA.

Since these stock data are annual, we estimate the quarterly values by using the Private Fixed Investment data taken from NIPA Table 5.3.5. Specifically, given the beginning-of-year stock and the end-of-year stock, the net addition to the stock during the year is distributed to each quarter depending on the share of the real investment for the quarter. This method is also adopted by Yogo (2005). The data are then shifted one period to convert to beginning-of-period values. Finally, the data is rescaled by dividing by the sample mean.

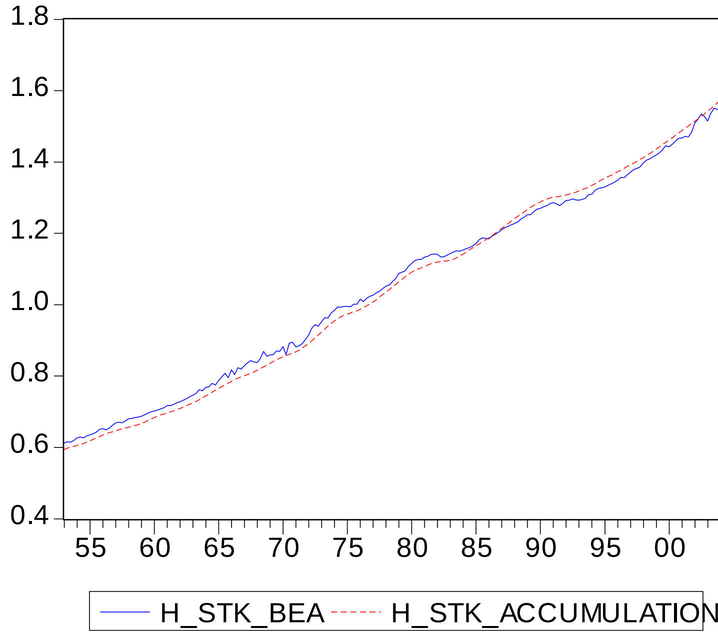
The second data is estimated by the accumulation equation:

$$H_{t+1} = (1 - \delta) H_t + I_t,$$

where H_t is real residential stock at the beginning of period t, I_t is real gross residential investment during period t, and δ is constant depreciation rate. For real investment, Real Residential Private Fixed Investment in NIPA Table 5.3.5 is used. The depreciation rate is set at 1.2% per year, or 0.2987% per quarter, which is roughly the average depreciation rate computed from the Detailed Data for Fixed Assets and Consumer Durable Goods from BEA. The initial value of residential stock is set so that the rescaled initial value matches the first data series.

As shown in Figure 3.2, two data series roughly match while the BEA-based data exhibits more short-term variability. We adopt the BEA-based data.

Figure 3.2: Estimated Housing Stock (Rescaled)



Asset returns: Market return is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks taken from Kenneth French's web site. Risk -free rate is three month constant maturity Treasury yield.

3.2.4 Results

Table 3.1 shows parameter estimates. Panel A shows the result for housing services data and Panel B shows the result for housing stock data. In each panel, two different types of the utility function (CES and GES), and two different versions of the pricing kernel ((3.-1) and (3.0)) are examined.

Table 3.1: Estimated Parameter Values of the CES/GES utility function.

Panel A: H_t = NIPA-based Housing Services
(1947:2-2006:3, $N = 238$)

	Pricing kernel 1			Pricing kernel 2		
	CES	GES		CES	GES	
ρ (SES)	6.009	0.850	***	4.241	0.912	***
	(4.014)	(0.027)		(1.610)	(0.021)	
θ (IES)	0.089	−0.080		0.128	0.140	
	(0.019)	(0.362)		(0.022)	(0.025)	
α (weight on housing)	0.400	2.001		0.372	0.561	
	(0.044)	(5.645)		(0.035)	(0.725)	
β (subjective discount factor)	1.035	1.029		1.018	1.023	
	(0.015)	(0.015)		(0.009)	(0.011)	
η (non-homotheticity)	–	0.841	***	–	0.922	***
	–	(0.026)		–	(0.017)	

Panel B: H_t =BEA-based Housing Stock
(1947:2-2004:1, $N = 227$)

	Pricing kernel 1			Pricing kernel 2		
	CES	GES		CES	GES	
ρ (SES)	0.813	0.379	***	0.731	0.409	***
	(0.104)	(0.054)		(0.102)	(0.055)	
θ (IES)	0.041	0.051		0.107	0.093	
	(0.013)	(0.016)		(0.028)	(0.024)	
α (weight on housing)	0.473	0.410	***	0.474	0.417	***
	(0.003)	(0.012)		(0.003)	(0.012)	
β (subjective discount factor)	1.111	1.094		1.034	1.039	
	(0.040)	(0.039)		(0.014)	(0.016)	
η (non-homotheticity)	–	1.217	***	–	1.179	***
	–	(0.041)		–	(0.036)	

Notes: In parentheses are Newey-West heteroschedasticity and autocorrelation consistent standard errors. Significance levels of *-10 percent, **-5 percent, and ***-1 percent represent whether point estimates for the CES function are outside of confidence interval of those for GES function.

The point estimate of SES (ρ) for the CES utility is quite large at 6.009 in Panel A (Column 1). It is consistent with the range reported by Davis and Martin [2005] ($\rho > 2.2$) although the estimate is imprecise. The estimates of SES diminish significantly to 0.850 when we allow for non-homotheticity by using the GES utility function. (Column 2) The estimate for GES is very precise and significantly below one at one percent level. The same pattern of reduced SES under non-homotheticity appears for the other specification. ($4.241 \rightarrow 0.912$ for pricing kernel 2 in Panel A) When we use housing stock data by assuming linear transformation of stock into service flows (Panel B), estimates of SES become lower. But still, upward bias for the CES specification is observed. ($0.813 > 0.379$ and $0.731 > 0.409$ in the first row in Panel B)

We like to test the null hypothesis that $\rho^{CES} = \rho^{GES}$, but there is no formal test statistic for these non-linear GMM estimators.[6] We examine three alternative statistics. The first is t-statistic for the null hypothesis that ρ^{GES} equals to the point estimate of ρ^{CES}. In other words, we treat ρ^{CES} as if it were non-stochastic. By using standard errors for ρ^{GES}, we reject the null hypothesis that $\rho^{CES} = \rho^{GES}$ at one percent level for any specification. The second statistic is an analogue of Hausman statistic: $S \equiv \left(\rho^{GES} - \rho^{CES}\right)^2 / \left(Var\left(\rho^{CES}\right) - Var\left(\rho^{CES}\right)\right)$. The statistics are 1.65 (PK1) and 4.28 (PK2) for Panel A, and 23.84 (PK1) and 14.05 (PK2) for Panel B. Although the appropriate degrees of freedom for the Chi-squared distribution are not obvious, the statistics are quite large for all specifications in a usual sense.[7] We fail to reject the null hypothesis of equality

[6] Hausman test utilizes the gap of two estimators with respect to consistency and efficiency. (e.g. OLS and IV estimators in testing endogeneity bias) Here, although ρ^{CES} and ρ^{GES} are distinct with respect to consistency under different hypotheses, there is no a priori difference in efficiency.

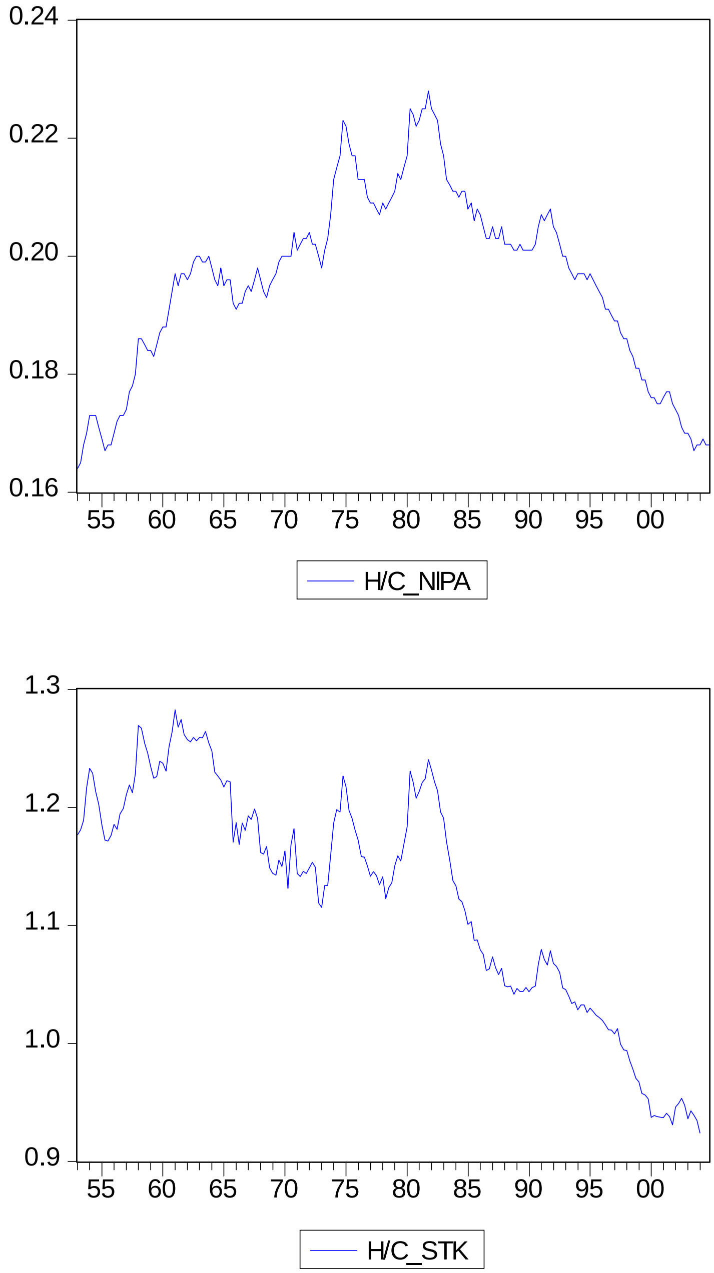
[7] In case of endogeneity bias, the number of potentially endogenous variables is the degrees of freedom. Here, RHS variables are common for both estimators.

at 1% level only if degrees of freedom are no less than $8, 14, 45$, and 30, respectively. We are likely to reject the null. The third statistic is the probability of $\rho^{CES} = \rho^{GES}$ based on bivariate normal distribution with zero correlation between them. The probability of equality is 0.044 (PK1) and 0.029 (PK2) for Panel A, and 0.004 (PK1) and 0.072 (PK2) for Panel B. Judging from three statistics above, it would be safe to conclude that the null hypothesis of $\rho^{CES} = \rho^{GES}$ is rejected.

This result of upward bias in SES casts an important caveat on empirical asset pricing with multiple goods. The CES utility function, or even more restrictive, the Cobb-Douglas and log-linear utility functions are workhorse in the literature. An upward bias in the estimates of SES caused by implicit assumption of homotheticity must be taken seriously.

The parameter for non-homotheticity (η) is 0.841 and 0.922 in Panel A. Both estimates are significantly different from one, supporting existence of non-homothetic property of preferences. As shown in Appendix C, $\eta < 1$ implies that income expansion is associated with a higher consumption share of housing. In other words, the result implies that income elasticity of housing demand is greater than that of non-housing demand. However, levels of the parameter are different if housing stock is used. (Panel B) Point estimates become 1.217 and 1.179, which are larger than one. As shown in Figure 3.3, historical patterns of consumption ratio of housing to non-housing substantially differ by data sets. Estimates of $\eta > 1$ for housing stock are driven by long-term decline in consumption ratio of housing. We further examine non-homothetic property of preferences in Section 3 by incorporating aggregate income explicitly in the OLS framework.

Figure 3.3: Consumption Ratio of Housing to Non-Housing



Estimates of IES (θ) are quite low for both CES and GES functions. The highest estimate is 0.140 with NIPA-based housing services and pricing kernel 2 for the GES function. Low estimates are consistent with the results from early studies.[8] However, our estimates of IES are likely to suffer from downward biases found in more recent studies. Bansal and Yaron [2004] argue that simple estimation of IES ignoring the effect of time-varying consumption volatility create significant downward bias on the estimate of IES. Vissing-Jorgensen and Attanasio [2003] also find a downward bias on estimates of IES when limited-stock market participation is not taken into account. They find IES of well above one after correcting for biases. If IES is greater than one after correcting for the downward biases, we have relative complementarity between two goods (i.e. IES>SES).

Estimates of α and β are around 0.45 and 1.03, respectively. Although subjective discount factor greater than one ($\beta > 1$) is violating standard assumption, it is a typical result from aggregate time-series data. (e.g. Yogo [2005])

3.3 Estimating SES with Linear Regression Models

3.3.1 Elasticities of Consumption Ratio and Housing Demand

Consider a static problem of a household with the CES-utility function: i.e. (3.2.1) with $\eta = 1$. The consumer's static problem is

$$\max_{C,H} u(C,H) \quad s.t. \;\; C + PH = Y, \tag{3.-1}$$

where P is relative price of housing services and Y is income.

[8]Among the large body of literature on the EIS estimation, Hall [1988] finds it to be negative and Yogo [2005] estimates it at 0.02.

Assuming an interior solution, the relative price of housing services and housing demand are, respectively,

$$P = \frac{\partial u/\partial H}{\partial u/\partial C} = \frac{\alpha}{1-\alpha}\left(\frac{H}{C}\right)^{-\frac{1}{\rho}}, \tag{3.0}$$

$$H = Y\left[P + \left(\frac{1-\alpha}{\alpha}P\right)^{\rho}\right]^{-1}. \tag{3.1}$$

Price and income elasticities of consumption ratio ($\varepsilon_P^{H/C}, \varepsilon_Y^{H/C}$) and housing demand ($\varepsilon_P^H, \varepsilon_Y^H$) at a particular price level are, respectively,

$$\begin{aligned}
\varepsilon_P^{H/C} &\equiv \frac{\partial \ln (H/C)}{\partial \ln P} = -\rho \leq 0, \\
\varepsilon_Y^{H/C} &\equiv \frac{\partial \ln (H/C)}{\partial \ln Y} = 0, \\
\varepsilon_p^H &\equiv \frac{\partial \ln H}{\partial \ln P} = -1 - \frac{(\rho-1)\left(\frac{1-\alpha}{\alpha}\right)^{\rho} P^{\rho-1}}{\left(\frac{1-\alpha}{\alpha}\right)^{\rho} P^{\rho-1} + 1} < 0, \\
\varepsilon_w^H &\equiv \frac{\partial \ln H}{\partial \ln Y} = 1,
\end{aligned}$$

Elasticities of consumption ratio ($\varepsilon_P^{H/C}$ and $\varepsilon_Y^{H/C}$) are particularly simple. Empirical relationship between log consumption ratio and log housing prices (i.e. $\varepsilon_P^{H/C}$) directly indicates SES.[9] We can also use coefficient on log income ($\varepsilon_Y^{H/C}$) for a test of homotheticity.

Price elasticity of housing demand (ε_p^H) is more complicated and depends on price level and utility share of housing. Although price elasticity is in general a non-monotonic function of SES, their levels are determined by whether SES is greater than or smaller than one. If SES is greater than one ($\rho > 1$), price elasticity of housing demand is also greater than one in absolute value. ($sgn\left(\left|\varepsilon_p^H\right| - 1\right) = sgn\left(\rho - 1\right)$). In this case, price elasticity of housing expenditure is negative: A higher price of housing services is associated with a lower expenditure on housing.

[9]Although this elasticity is for Marshalian (uncompensated) demand, income compensation will not alter the consumption ratio under the CES utility assumption.

We can derive a reduced-form empirical equations based on the CES model above. By taking logs of (3.0) and differentiating, we obtain $d\ln(H/C)_t = -\rho d\ln P_t$.By allowing for the intercept and effect of income, we obtain an empirical equation of consumption ratio:

$$d\ln(H/C)_t = a + bd\ln P_t + cd\ln Y_t + \mu_t,$$

where μ_t are disturbances. $a = c = 0$ and $b = -\rho$ according to the CES model. Estimating consumption-ratio equation is relatively unique in this research. Vast majority of previous studies focus on housing demand and expenditure for the sake of demand analyses of housing. In the current research, our focus is to obtain SES for the sake of asset pricing analysis.

Furthermore, we allow for the possibility that different consumers have different parameter values in their utility functions. We focus on the source of household income. Income distribution among different types of consumers affects aggregate demand unless income expansion paths for all consumers are parallel straight lines.[10] We simply hypothesize that the economy consists of two types of representative consumers: investors and employees. If investors and employees have different SES, we expect to observe time-variations in consumption ratio as composition of two types of consumers change over time. The model is shown in Appendix D. We empirically examine whether income share of non-employee income affects consumption ratio of housing, independent of income levels, by adding a term of income share of investors:

$$d\ln(H/C)_t = a + bd\ln P_t + cd\ln Y_t + dd\ln w_t^i + \mu_t. \tag{3.-3}$$

[10]The condition is equivalent to indirect utility function of any consumer k having the Gorman form: $v_k(p, Y_k) = a_k(p) + b(p)Y_k$.

After obtaining the sign of coefficient d in equation (3.3.1), we can infer relative level of SES for the two types of consumers by assuming a common value of α. Although α cannot be identified in this regression, it is estimated in the GMM estimation in the previous section. As shown in Appendix 3, if $P\left(1-\alpha^k\right)/\alpha^k > 1$ for $k = e, i$, a positive value of coefficient d implies that investor has a lower SES than employee does ($\rho^i < \rho^e$), and vice versa.

We also estimate price and income elasticities of housing demand. By taking logs of (3.1) and differentiating, we obtain $d\ln H = \varepsilon_p^H d\ln P + \ln Y$. A simple way to reflect price elasticity being a function of price is to include a term of squared log price in empirical equation. Together with an income share term,

$$\begin{aligned} d\ln H_t &= a + (b_0 + b_1 \ln P_t)\, d\ln P_t + cd\ln Y_t + dd\ln w_t^i + \mu_t \\ &= a + b_0 d\ln P_t + b_1 (d\ln P_t)^2 + cd\ln Y_t + dd\ln w_t^i + \mu_t. \end{aligned} \tag{3.-3}$$

According to the CES model, $a = 0, b_0 < 0, b_1 \leq 0$ and $c = 1$.[11]

By adding $d\ln P_t$ to both sides of equation (3.-3), we can obtain an empirical equation of housing expenditure in log-linear form. Although previous studies often derive linear expenditure equations from the Stone-Geary utility function, the linearity of expenditure comes at the expense of restrictions on SES in the Stone-Geary utility function. We will examine the Stone-Geary utility function in the next section.

3.3.2 Data

In addition to the data set used in the GMM estimation in Section 2, we use aggregate data for income and income share of investors.

[11] Since price elasticity is negative, $b_0 + b_1 \ln P_{H,i} < 0$. In addition, $\partial\varepsilon_p^H/\partial P_H = -(\rho-1)^2\left(\frac{1-\alpha}{\alpha}\right)^\rho {P_H}^{\rho-2}\left[\left(\frac{1-\alpha}{\alpha}\right)^\rho {P_H}^{\rho-1}+1\right]^{-2} \leq 0$. Therefore $b_0 < 0$ and $b_1 \leq 0$.

Income level: We use per capita real GDP as an income measure. GDP is current income, which includes both permanent and transitory income. For our purpose to examine homotheticity, current income is relevant since it is well-known that households also respond to transitory-income shocks. Although some of previous studies use total consumption as a proxy for permanent income, use of total consumption as income level causes a econometric problem in our regression of consumption ratio. Log non-housing consumption, which enters negatively on the left-hand side, is a major source of variations in total consumption, which enters positively on the right-hand side. Therefore, a clear negative relationship between consumption ratio and "permanent income" is observed if we use total consumption as income proxy.

Income share of investors: Employees' income is "Compensation of employees, received" (line 2) in NIPA Table 2.1-Personal Income and Its Disposition, published by BEA. Investors' income is the sum of 1) Proprietors' income with inventory valuation and capital consumption adjustments (line 9), 2) Rental income of persons with capital consumption adjustment (line 12), and 3) Personal income receipts on assets (line 13), in the same table. The income share of investor is investors' income divided by the sum of both types of income.

As a preliminary check of time-series properties of the data, we first test unit-root by Augmented Dickey-Fuller test with four lags and with intercept and trend terms. All series are $I(1)$; the null hypotheses of unit-root are not rejected for levels but are rejected for first-differences at one percent level. Second, we check for cointegration by Johansen's LR test. For each of consumption ratio equation, housing demand equation and housing

Table 3.2: Estimated Elasticities of Consumption Ratio. Dependent Variable = $\text{dln}(H_t/C_t)$.

Panel A: H_t = NIPA-based Housing Services (1953:2-2006:4, $N = 215$)

	(1)		(2)		(3)	
Constant	–		0.003	***	0.003	***
	–		(0.001)		(0.001)	
$d\ln$ (Housing Prices)	−0.682	***	−0.617	***	−0.617	***
	(0.138)		(0.090)		(0.084)	
$d\ln$ (Income Level)	–		−0.511	***	−0.521	***
	–		(0.065)		(0.065)	
$d\ln$ (Income Composition)	–		0.083	**	0.080	**
	–		(0.038)		(0.040)	
d (10y Treasury Yield)	–		–		0.001	
	–		–		(0.001)	
Adjusted R^2	0.182		0.454		0.461	
DW	1.68		2.02		2.09	

Panel B: H_t =BEA-based Housing Stock (1953:2-2004:1, $N = 204$)

	(1)		(2)		(3)	
Constant	–		0.002	**	0.002	**
	–		(0.001)		(0.001)	
$d\ln$ (Housing Prices)	−0.554	***	−0.445	***	−0.474	***
	(0.141)		(0.100)		(0.111)	
$d\ln$ (Income Level)	–		−0.500	***	−0.485	***
	–		(0.075)		(0.080)	
$d\ln$ (Income Composition)	–		0.064		0.074	
	–		(0.072)		(0.076)	
d (10y Treasury Yield)	–		–		−0.001	
	–		–		(0.001)	
Adjusted R^2	0.081		0.276		0.276	
DW	1.99		2.52		2.52	

Notes: Significance levels: *: 10 percent; **: 5 percent; ***: 1 percent. In parentheses are Newey-West heteroschedasticity and autocorrelation consistent standard errors.

expenditure equation, any cointegration is rejected at 5% level. Therefore, we use log-differenced series for estimation.

3.3.3 Result

Table 3.2 shows the estimation result of consumption ratio equation (3.3.1). Panel A is based on housing services data taken from NIPA and Panel B is based on housing stock data. In each panel, we estimate three different versions. In the third specification, we include 10-year Treasury yield in order to account for potential intertemporal effects on consumption ratio. 3-month Treasury yield generates almost identical results, which is not reported for brevity.

The price elasticity, or negative of SES, is -0.617 with NIPA-based housing services data, and between -0.445 and -0.474 with BEA-based housing stock data. All estimates are significantly below one at one percent level. Overall level of coefficient is roughly consistent with the results from our GMM estimation: SES is well below one by allowing for non-homotheticity, and is lower when we use housing stock data. We conclude that SES between housing and non-housing is below one between 0.4 and 0.85 once we allow for non-homothetic preferences.

The income elasticity of consumption ratio is about -0.5 for all specifications, and is significantly less than zero at one percent level. A higher income is, ceteris paribus, associated with a smaller share of housing consumption. Although we obtain mixed results on signs in our GMM estimation, here we obtain consistently negative estimates for both data sets. The result indicates that consumers have non-homothetic preferences, and in particular, income elasticity of housing demand is lower than that of non-housing. This is consistent with observed pattern that the ratio of housing expenditure to income is decreasing in income levels, as reported by Green and Malpezzi [2003].

The coefficients on income composition (i.e. income share of investors) are positive, and statistically significant at 5 percent level when we use housing services data. The consumption share of housing increases as income is derived more from investments. The GMM estimates of α are about 0.41 with the GES specification. Then $P\left(1-\alpha\right)/\alpha > 1$ in sample, and a positive coefficient implies that investors have a lower SES than employees. If employees are more likely to be renters, then the result is consistent with the findings in the previous literature that renters exhibit a higher price elasticity of housing demand.

Interest rate does not affect consumption share significantly. Possibly, appropriate interest rates are incorporated into market rent and interest rate itself does not have a direct effect on housing demand.

Statistical fit and properties of residuals are much better for the consumption ratio equation than housing demand equation (Table 3). For example, adjusted R-squared is 0.461 and the Durbin-Watson statistic is 2.09 when housing service data is used. These statistics are much lower for the housing demand equation.

Table 3.3 shows the estimation result of housing demand equation (3.-3). For potential endogeneity bias caused by simultaneous equations of supply and demand, we conduct Wu-Hausman endogeneity test. The instrumental variables of the supply-side equation is construction cost and housing starts. Data for construction cost is Price of Residential Investments taken from NIPA Table 1.5.4, deflated by non-housing price index.

Table 3.3: Estimated Elasticities of Housing Demand

Dependent Variable = $d \ln H_t$
H_t = NIPA-based Housing Services
(1953:2-2006:4, $N = 215$)

	(1)		(2)	
Constant	0.006	***	0.006	***
	(0.001)		(0.001)	
$d \ln$ (Housing Prices)	−0.310	***	−0.306	***
	(0.057)		(0.056)	
$d \ln$ (Housing Prices)^2	0.308		−0.066	
	(3.552)		(3.566)	
$d \ln$ (Income Level)	0.083	***	0.076	**
	(0.030)		(0.031)	
$d \ln$ (Income Composition)	0.030		0.028	
	(0.024)		(0.024)	
d (10y Treasury Yield)	–		0.000	
	–		0.001	
Adjusted R^2	0.200		0.191	
DW	0.94		0.94	

Notes: Significance levels: *: 10 percent; **: 5 percent; ***: 1 percent. In parentheses are Newey-West heteroschedasticity and autocorrelation consistent standard errors.

Housing starts are New Residential Construction for Total privately owned, published by the U.S. Census Bureau. We strongly reject the null that the endogeneity problem affects the estimates. Therefore, we use OLS rather than 2SLS for the sake of efficiency.

The mean price elasticity is about -0.3 based on the mean value of $d\ln$(Housing Prices). It is significantly greater than -1 at one percent level. Although there is no closed form solution for ρ in terms of ε_P^H, we can conclude that ρ is significantly below one. We obtain the same result of a low SES as in GMM estimation. In the previous studies using cross-sectional regressions, the price elasticity range from -0.3 to -0.8 (e.g. Mayo [1981]) Our result is at the low end of the range in absolute value. Price elasticity of less than one in absolute value implies that housing expenditure is greater when price of housing services is high, which agrees with our casual observation.

Income elasticity of housing demand is very low about 0.08. A low income elasticity of housing demand is consistent with negative income elasticity of consumption ratio reported in Table 2. It is also consistent with the results from previous studies that income elasticity is well below one. However, previous estimates of income elasticity range from 0.3 to 0.8. Our result is far below previous estimates.

Income composition and interest rate do not enter significantly; the same result as in consumption ratio estimation.

3.4 Linear Expenditure Equation Based on the Stone-Geary Utility Function

Another departure from the plain CES utility function for non-homothetic preferences is introducing subsistence levels. The Stone-Geary utility function is a modified Cobb-Douglas utility function with subsistence levels:

$$U(C,H) = \left(C - \bar{C}\right)^{1-\alpha} \left(H - \bar{H}\right)^{\alpha}, \; C > \bar{C}, \; H > \bar{H}, \tag{3.-5}$$

where $\bar{C}$, $\bar{H}$, are subsistence levels of C and H, respectively, and $\alpha \in (0,1)$.

Demand for housing services (H) from the static problem (3.3.1) is

$$H = \frac{\alpha\left(Y - \bar{C}\right)}{P} + (1-\alpha)\bar{H}. \tag{3.-5}$$

The demand is the sum of the minimum level of housing consumption ($(1-\alpha)\bar{H}$) and the usual demand with Cobb-Douglas utility except for income adjustment for minimum level of non-durable consumption ($\alpha\left(Y - \bar{C}\right)/P$). Housing expenditure is written as a linear function of income and price:

$$PH = -\alpha\bar{C} + \alpha Y + (1-\alpha)\bar{H}P.$$

We specify empirical equation for housing expenditure by including income share of investors:

$$P_t H_t = a + bP_t + cY_t + dw_t^i + \mu_t. \tag{3.-5}$$

Then coefficients a, b and c just identify parameters $\alpha, \bar{C}$ and $\bar{H}$.

The price elasticity of housing demand ε_P^H is

$$\varepsilon_P^H \equiv \frac{\partial H}{\partial P}\frac{P}{H} = -\frac{\alpha\left(Y - \bar{C}\right)}{PH} = -\left[1 + \frac{(1-\alpha)P\bar{H}}{\alpha\left(Y - \bar{C}\right)}\right]^{-1}. \tag{3.-5}$$

If there is no subsistence levels in housing consumption (i.e. $\bar{H} = 0$), price elasticity of housing demand is negative one as with simple Cobb-Douglas utility function. With $\bar{H} > 0$, price elasticity is strictly less than one in absolute value ($\left|\varepsilon_P^H\right| < 1$). This is understood by recognizing that price elasticity of $H - (1-\alpha)\bar{H}$ is always negative one, independent of price or income.[12] Necessary amount of response of $H - (1-\alpha)\bar{H}$ is obtained by a smaller response of H.

Regarding homotheticity, income elasticity of consumption ratio is shown to be

$$\begin{aligned}
\varepsilon_Y^{H/C} &\equiv \frac{\partial \ln (H/C)}{\partial \ln P} \\
&= \alpha \left(\frac{PH}{Y}\right)^{-1} - (1-\alpha)\left(\frac{C}{Y}\right)^{-1} \\
&= \left[\alpha \left(\frac{P\bar{H}}{Y}\right)^{-1} - (1-\alpha)\left(\frac{\bar{C}}{Y}\right)^{-1}\right] \frac{\bar{C}P\bar{H}}{(Y-C)\,C}.
\end{aligned}$$

The sign of income elasticity of consumption ratio is determined by

$$\begin{aligned}
sgn\left(\varepsilon_Y^{H/C}\right) &= sgn\left(\frac{\alpha}{1-\alpha} - \frac{PH}{C}\right) \\
&= sgn\left(\frac{\alpha}{1-\alpha} - \frac{P\bar{H}}{\bar{C}}\right). \qquad (3.\text{-}8)
\end{aligned}$$

Again, if there is no subsistence level (i.e. if the utility function is simple Cobb-Douglas one), income elasticity of consumption ratio is zero since α and $1-\alpha$ are just expenditure share of housing (PH/Y) and non-housing (C/Y), respectively. If expenditure share of housing is greater than α, or equivalently, minimum housing expenditure $\left(P\bar{H}\right)$ is sufficiently large relative to minimum non-housing expenditure $\left(\bar{C}\right)$, income elasticity of consumption ratio is negative: Consumption share of housing decreases as income increases.

Empirically, after estimating parameters of the Stone-Geary utility function $(\alpha, \bar{C}, \bar{H})$

[12] This is easily confirmed from (3.4).

from the linear expenditure equation, we can infer the sign of income elasticity of consumption ratio by combining with average price of housing services.

Table 3.4 shows estimation result of linear expenditure equation (3.4). in estimating this equation we use $C_t + P_t H_t$ as a measure of income level in order to satisfy static budget constraint. Coefficients for d (Housing Prices) and d (Income Level) are BLUE for $(1-\alpha)\bar{H}$ and α, respectively. We calculate the sample mean of

$$P_t H_t - \left[\begin{array}{c} 0.011t + 2.072\,(Housing\ Prices)_t \\ +0.012\,(Income\ Level)_t + 0.253\,(Income\ Composition)_t \end{array} \right],$$

and obtain the estimate of $-\alpha\bar{C} = -1.011$ with standard deviation of 0.039. Then, our estimates of parameter values are $\alpha = 0.031, \bar{C} = 32.990, \bar{H} = 1.917$, all of which are significantly different from zero at one percent level. Non-zero subsistence levels of consumption implies again non-homothetic preferences.

Using our parameter estimates, we obtain $P\bar{H}/\bar{C} = 0.052 > 0.032 = \alpha/(1-\alpha)$. We infer from (3.-8) that income elasticity of consumption ratio ($\varepsilon_Y^{H/C}$) is negative; i.e. consumption share of housing declines as income grows, also with the Stone-Geary utility function. However, estimates of $\bar{C}$ and $\bar{H}$ are unreasonably high due to low value of α, given that the means of C and H are 13 and 2.5, respectively. Low α is obtained because housing expenditure ($P_t H_t$) is much smoother than income level ($C_t + P_t H_t$). For instance, if $\alpha = 0.1$, $\bar{C}$ and $\bar{H}$ become 10.1 and 2.1, respectively.

Table 3.4: Estimated Elasticities of Housing Expenditure

Dependent Variable = $d\,(P_t H_t)$
H_t = NIPA-based Housing Services
(1953:2-2006:4, $N = 215$)

	(1)		(2)	
Constant	0.010	***	0.010	***
	(0.001)		(0.001)	
d (Housing Prices)	1.859	***	1.909	***
	(0.222)		(0.227)	
d (Income Level)	0.031	***	0.029	***
	(0.001)		(0.006)	
d (Income Composition)	0.293		0.259	
	(0.209)		(0.214)	
d (10y Treasury Yield)	–		0.001	
	–		(0.002)	
Adjusted R^2	0.613		0.615	
DW	1.11		1.12	

Notes: Significance levels: *: 10 percent; **: 5 percent; ***: 1 percent. In parentheses are Newey-West heteroschedasticity and autocorrelation consistent standard errors.

3.5 Conclusion

Relative size of the IES compared to the SES between different goods is important in asset pricing. In estimating the IES and SES, moment conditions are usually derived from a utility function with homothetic property, such as log-linear, Cobb-Douglas, or CES-power utility function. However, if the true preferences of consumers are non-homothetic, the homotheticity assumption is likely to generate a bias on the estimates of SES because any change in consumption ratio is associated to a price change.

In this research, we estimate the IES and the SES for housing by allowing for non-homothetic preferences. Although very high estimates of SES are obtained in previous research that uses the CES-power utility function, we show that estimates become much lower when non-homotheticity is allowed. We confirm an upward bias in estimates of SES when the homotheticity assumption is imposed. Since the CES-power utility function is a most commonly used form in empirical asset pricing, it is very important to notice the bias when we interpret empirical results.

In our results, SES is well below one, ranging from 0.4 to 0.9. Homotheticity is strongly rejected in such a way that income growth leads to a smaller consumption weight on housing. The result is consistent with anecdotal evidence that households tend to spend more on housing in areas with high housing costs. It is also consistent with previous estimates of price and income elasticities of housing demand, both of which are less than one. Estimates of IES are very low around 0.1. Although the result implies that housing and non-housing are relatively substitutable ($\theta - \rho < 0$), our estimate of IES may be subject to a downward bias caused by time-varying consumption volatility and other factors. After correcting for the bias on IES, we may well obtain the IES greater than one. Correcting explicitly for the bias on IES is a future task.

Another extension is to apply the same procedure to other countries. Since we mainly use aggregate time-series data from the national income account, data is easily available. Using a different measure of housing price might be also fruitful. The NIPA-based PCE price index, which we use in the current research, should be equal to the user cost of housing in a frictionless economy, but estimated user cost measures are usually more volatile. More volatile price measure may result in lower estimates of SES.

Chapter 4

Strategically Delayed Investments in Joint Projects

4.1 Introduction

This chapter studies the investment decision of agents when a small number of agents form a joint project, in which strategic concerns play an important role. Investment in a strategic environment has been studied mainly in the context of preemption and entry deterrence (e.g. Dixit [1979] for the certainty case). The standard result is that investments are accelerated in the strategic environment because early investments will limit competition by deterring potential entrants. Even under uncertainty, when strategic effects are introduced, investments are accelerated relative to the monopoly case since the investor trades off the benefits of delaying and preempting (e.g. Grenadier [2002]).

However, we often observe that investments are delayed rather than accelerated in joint projects. The members of the project eventually commit themselves (i.e. make irreversible investments) since the joint project, if completed, will provide a positive gain. However, due to the risk in the project value, the members also have motives not to commit themselves too much and to remain flexible, especially in the early stage. Therefore, there is a trade-off between commitment and flexibility in this economic environment.

Joint projects formed by a contract of multiple parties are in fact an essential basis of the economic activity. At the individual level, a labor contract can be seen as a joint project of the worker and the firm. For example, a professional pianist and an impresario make a contract for a concert.[1] A venture firm founded by a technician and a professional manager is another example. At the firm level, cross-sectional task forces are often formed. At the industry level, large scale projects such as R&D and urban development are often conducted by the alliance of multiple firms.[2] Alliances for the new DVD format and the redevelopment of former industrial sites are good examples. At the multinational level, investments by international organizations are a joint project of the member countries that require investments by all member countries.

In this chapter I show that, when strategic effects exist, the equilibrium level of flexibility built in the initial contract is greater, and that the investment timing is delayed than in the cases without such effects. The model takes into account, not only the effect of uncertainty on the choice of flexibility, but also the reverse effect of keeping flexibility on uncertainty. Flexibility creates endogenous uncertainty through the strategic interactions

[1] This example is taken from Hart and Moore [2004].

[2] Large-scale urban renewal projects by means of public-private partnerships have often been conducted in Europe and Japan. In the U.S., the judgment by the Supreme Court on the recent case between the city of New London, Connecticut, and its residents is likely to stimulate the formation of such forms of development in the future. The Court provided the strong affirmation to the use of eminent domain for economic rejuvenation.

among agents. While the benefit of keeping flexibility in response to the exogenous uncertainty is widely studied, the reverse channel from flexibility to uncertainty is new in this chapter.

Positive feedback between total uncertainty and the degree of flexibility is the key mechanism. Positive feedback starts from exogenous uncertainty such as demand uncertainty, which rationalizes the choice of a flexible plan by each party. However, the flexibility kept by each party creates strategic uncertainty on the other parties through synergies. Taking account of both demand uncertainty and strategic uncertainty, the remaining parties choose a higher degree of flexibility as their best response. This higher degree of flexibility poses higher strategic uncertainty to other parties.

A simple model of two firms, in which an option model of investment is embedded in a stage game, is built to derive the Nash equilibrium. After modeling the value of investment opportunity as an American call option, I show that the flexibility is a strategic complement as defined by Bulow et al. [1985]. That is, a party's higher degree of flexibility induces the other to choose a higher degree of flexibility. Then I show that a more flexible choice by a party has a feedback effect to increase its own uncertainty. A higher degree of flexibility and delayed investment are derived in equilibrium. The results contrast to the previous models of strategic investment, in which the strategic factor induces firms to invest early. The current model predicts stagnant investment activities without reduced demand. The model seems to explain well the various cases of joint investments in new technologies to the public-private partnership in urban renewal. This model can also serve as a basis of the status quo bias in the political economy.

This chapter is complementary to the existing literature in two aspects of economic environment. The first is the competitive environment. This chapter deals with investments in a joint project, in which the parties are not competing each other in the same market. In contrast, the literature on strategic investment in industrial organization and real options deals with the competition of a few firms in the same market.[3] In the industrial organization literature, Jones and Ostroy [1984], Appelbaum and Lim [1985], Vives [1989] and Spencer and Brander [1992] consider trade-offs between flexibility and pre-commitment under such an economic environment.

The second aspect is the source of flexibility. This chapter considers the investment decision as the source of flexibility when other dimensions of flexibility (e.g. product choice) are allowed at the same time. In contrast, product-differentiation models treat the product choice as flexible when firms compete each other in the close markets. Most directly complementary to this chapter is Hart and Moore [2004], which studies the flexible product choice in the contractual joint project.

Therefore, in the matrix of competitive environment= {compete, cooperate} and source of flexibility= {investment decision, product choice}, this chapter is positioned in the investment-cooperate cell. Strategic real option models and preemptive investment models are in the investment-compete cell, product differentiation is in the product-compete cell, and Hart and Moore [2004] is in the product-cooperate cell. This is shown in Figure 4.1.

The chapter is organized as follows. Section 2 introduces two examples. In Section 3, I review the existing literature in order to characterize this chapter. Section 4 describes

[3] Kogut [1991] and Chi [2000] study joint venture, but the real-option intuition is used to consider the decision to acquire (terminate) the joint venture.

Figure 4.1: Characterization of the paper.

		Source of Flexibility	
		Investment decision	Product choice
Competitive Environment	Compete	Preemptive Investment, Strategic Real Options	Product Differentiation
	Cooperate	**This paper**	Hart & Moore (2004) (Mechanism Design)

the model of investment decision, and Section 5 analyzes the equilibrium choices of flexibility and timing of investment. Section 5 concludes.

4.2 Examples

4.2.1 Urban renewal in Japan

After the "bubble burst" of the real estate market in 1990, real estate prices continually dropped throughout the 1990's. During 1990's, most real estate development projects were stalled and urban renewal did not make much progress.

In 2001, the Cabinet Secretariat of Japan established the Urban Renaissance Headquarter (URH), a new organization to stimulate the urban renewal. Due to the severe fiscal problem of the Japanese government, the URH tried to rely much on the private development so that it would not increase the government budget.

The hope for private initiatives was not totally unreasonable since the private sector's damage from the low real estate price was greatly relieved until that time by the debt write-offs by the financial institutions. The URH considered reshaping the development process, particularly in some selected geographical areas. The Priority Urban Redevelopment Areas (PURA) were chosen to be developed by public-private partnerships. The government announced to give special treatment in the development approval process in return for the provision of capital by the developer for the projects. If both the government and the firms had made project-specific investments to complete the projects in PURA, the projects should have provided benefits to both the government and the private firms. However, the program did not jump-start and urban renewal remained stagnant.

There were substantial uncertainty in the actions of both URH and private developers. Since the program was an unprecedented one, the details of the investment plan and the role-sharing rule were not determined when the program started. The action of the government was in particular unpredictable, but there was also uncertainty about whether the firms were capable of completing the projects under financial constraint.

Faced with these uncertainties, both parties tried to keep flexibility and hesitated to commit themselves. Given the monopoly position in investment opportunity, keeping more flexibility was a rational strategy. However, due to the sustained flexibility by both parties, they were faced with more uncertainty. An individually rational choice of flexibility generated strategic uncertainty. With this strategic uncertainty, the Nash-equilibrium level of flexibility is higher and the progress of the project became slow.

4.2.2 Alliances for the new DVD format

As the next-generation DVD format, there are two alternative technologies: Blu-ray Disc proposed by Blu-ray Disc Association (BDA) and HD-DVD proposed by DVD forum. Blu-ray Disc, which was proposed in February 2002 by Sony and other manufacturers, uses a blue laser to increase the capacity to 25 GB per layer. However, this presents significant manufacturing problems and requires new mastering and replication equipment and processes. HD-DVD, which was originally called AOD when it was launched in August 2002 by Toshiba and NEC, provides a smaller capacity of 10-20GB, but can be manufactured using existing DVD lines and existing mastering equipment.

Both camps have been trying to bring in as many hardware and software providers as possible. As of April 2005, over 100 companies support Blu-ray Disc and 83 companies support HD-DVD. However, some companies such as Apple Computer Inc. keep their wait-and-see attitude and express support for both camps. Apple's attitude seems to represent the attitudes of most firms since only core firms such as Sony in the Blu-ray Disc camp and Toshiba in the HD-DVD camp have been making significant investments.

Coexistence of both formats will significantly increases the cost of using the new-generation DVD technologies for consumers and limit the total market size. In April 2005, both camps announced to try to unify the formats by early May. May 2005 is really the deadline for the unification when the production schedule of the first-model machines is taken account of. Therefore, the essential investment for standardization was in fact postponed until the deadline. It took more than three years until this essential investment is made.

This whole process for the single format, rather than R&D process in each camp, can be seen as the true joint project of all involved firms. Although there is no formal contract for this joint project, all manufacturers and contents providers have been aware that they are involved in the standardization process of the new DVD format. Due to the technological immaturity, two technological alternatives were on the initial list in 2002. Given the flexibility in the choice of technology, strategic uncertainty emerged. Strategic uncertainty depressed the commitment motive of the firms and made the investment postponed until the deadline.

4.3 Existing Literature

In this section, I characterize this chapter by reviewing the existing literature. After a brief review of option-based investment models, three categories of literature in Figure 4.1 are reviewed: Strategic investment models (i.e. preemptive investment in industrial organization and strategic real option models), product differentiation models, and Hart and Moore [2004] in mechanism design.

4.3.1 Option-based models of investment

The optimal investment policy has been studied in a large body of literature. In the standard neoclassical framework, the criterion is Tobin's q, or the ratio of the market value of the asset to the replacement cost, being greater than one (Tobin [1969]). The theory, however, is augmented by taking account of the option value of investment opportunity arising from irreversibility.[4]

In the augmented framework, Tobin's q criterion works if there is no option value in investment opportunity, that is, if the investment cannot be delayed, the investment is

[4] One of the early literature on irreversible investment is Arrow [1968]. An often cited work is Bernanke [1983]. The early literature on the option model of investment includes Titman [1985], Brennan and Schwartz [1985], McDonald and Siegel [1986], Dixit and Pindyck [1994] and Abel et al. [1996].

reversible or the investment opportunity erodes by competition[5]. Otherwise, investment decision must take account of the opportunity cost of discarding the valuable options to invest later. That is, the option-based model must be used if the investment can be delayed and if the investment is not reversible and if the investor has some non-competitive position in the investment opportunity[6]. The implication is that the larger the option value of investment opportunity is, the higher the hurdle for the immediate investment becomes.

Basic properties of investment option

The option to invest can be valued, in principle, in the same way as the financial call option. The call option is the right without obligation to buy the underlying asset at a pre-specified price at the expiration date (European call) or at anytime before the expiration date (American call). For an investment project, the option is the right to obtain the risky asset by paying replacement cost at a preferred timing.

Option value has the unique feature that it is increasing in the volatility of the underlying asset. In general, an option is a function of the value of the underlying asset, exercise price, time to expiration, the risk-free rate of interest and the volatility of the underlying asset. The positive relation between the option value and the risk of the underlying asset results from the convexity of option's payoff function. Since the payoff function of an option is convex in the value of underlying asset, the Jensen's inequality term adds value to the expected present value of the option if there is risk in the value of the underlying asset.

The flexibility in the investment plan is nothing but the state contingency. A

[5]This third point is articulated by Grenadier [2002].

[6]A monopoly or oligopoly market is an obvious example. Novy-Marx [2005] considers the limited competition arising from endogenous heterogeneity in firms' capacity.

more flexible investment plan contains more options. Therefore, a flexible investment plan is thought of as a bundle of more than one option. The value of such a bundle of options is a non-decreasing function of the number of options because the value of each option is non-negative. Therefore, a more flexible plan has a greater value, omitting the cost of obtaining options.

4.3.2 Strategic investment

The literature about strategic investment focuses on the competition of firms in the same market rather than cooperative activities of firms. There are two strands of models: preemptive investment models in industrial organization and real option models with strategic interaction. In both strands, to the best of my knowledge, existing models focus on the effect of exogenous uncertainty on the choice of flexibility. One of the innovations is to incorporate the reverse effect: The effect of flexibility on uncertainty.

Preemptive investment in industrial organization

In the theory of industrial organization, commitment has been studied in the context of preemptive investment and entry deterrence (Dixit [1979]). When firms potentially compete in the same market, the first mover can prevent another firm from entering the market by committing itself to its continued operation to form a credible threat for a fierce competition. Discarding one's flexibility is a good strategy in this context. The underlying idea is the strategic move defined by Schelling [1960]: "One constrains the partner's choice by constraining one's own behavior."

When uncertainty is introduced, a trade-off between commitment and flexibility emerges. Although the existing literature on commitment and flexibility considers various types of flexibility, uncertainty, and trade-offs, a robust result derived in these models is that greater uncertainty favors the choice of flexible positions and discourages preemption (Jones and Ostroy [1984], Appelbaum and Lim [1985], Vives [1989], and Spencer and Brander [1992]). This is consistent with the standard result from real option models.

Strategic real options

The recent real options literature deals with strategic interaction between agents.[7] For example, Weeds [2002] considers strategic interaction in R&D competition and shows that, not only the leader-follower structure with usual preemption motive, but also a collusive outcome are supported in equilibrium.

It is important to note that option value of investment opportunity does not always exist. When investment options at the industry level is considered, whether the option value exists or not critically depends on the market structure. Grenadier [2002] shows that the competition among firms erodes the value of options and that ignoring the option value is the appropriate investment decision. In contrast, Novy-Marx [2005] shows that the heterogeneity of firms in capacity emerges endogenously under general assumptions on production technology so that the competition is effectively limited. Then the option value of investment opportunity remains significant. In this chapter, a joint project is defined by the initial contract and the investment opportunities are exclusive to the parties to the contract.

[7] A good survey is Smit and Trigeorgis [2004].

4.3.3 Product differentiation

When products can be differentiated, products are close substitutes but are not identical. Firms compete by choosing their product so that they can enjoy rents from limited competition. In spatial models of product differentiation, firms compete locally and choose the characteristics of their products in either linear or circular space. In monopolistic competition models, a product competes with all other imperfect substitutes in the market (Chamberlin [1933], Dixit and Stiglitz [1977], Hart [1985]). These models are located in opposite from the current model in Figure 4.1 in the sense that the firms compete in the same market and makes their choice of product instead of investment decision.

4.3.4 Mechanism design

The choice of flexibility is dealt with by some recent research on mechanism design, among which Hart and Moore [2004] is most related to this chapter. They study the contract that specifies the list of all the possible outcomes from their transaction ("rule out" in their terminology). The parties to the contract bargain later to choose one of the outcomes on the list ("rule in"). In other words, flexibility is built in the contract. They consider, like in this chapter, the trade-off between the merit of flexibility (i.e. state contingency) and the disadvantage of flexibility. The disadvantage is the possibility to have an unfavorable outcome by ex-post bargaining. They show that the equilibrium level of flexibility depends on the level of project-specific investment: a small specific investment leads to a more flexible contract. Note that they implicitly assume that the contract ensures the parties a monopoly position in the project so that Grenadier [2002]'s result does not apply (as in the present model).

The economic environment of this chapter is quite similar to the one in their model.[8] In the current model, a joint project is formed by the contract that allows flexible ex-post choice of investment plan. The distinction is that they allow flexible choice of product but fix the timing of project-specific investment while I allow flexible choice of product (or technology) AND flexible timing of investment. Therefore, I consider the equilibrium timing of investment in addition to the equilibrium choice of initial flexibility.

4.4 The Model

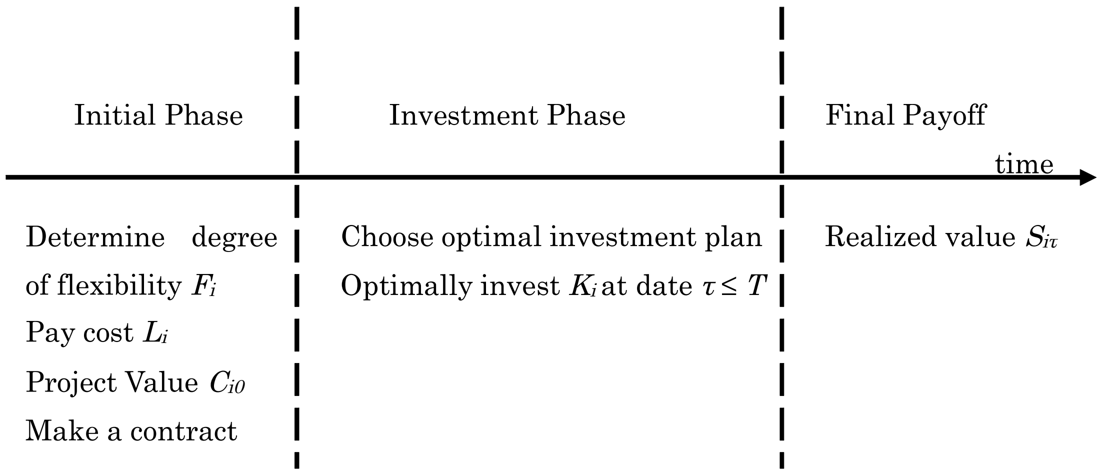
Two risk-neutral firms labeled $i = \{1, 2\}$ form a joint investment project, which requires investments by both firms. Either firm alone cannot make the whole investment because of limited capacity and required technology. The firms reach an agreement to form a joint venture only if both firms expect to realize positive net present value from their investments. The joint venture has a monopolistic position on the investment opportunity to some extent so that the timing of investments can be chosen.

The firms complete the project in two phases: Initial phase of negotiation to form a joint venture, followed by the second phase of investments. (See Figure 4.2.) In the negotiation phase, firms determine their joint investment plans, but they cannot contract on every potential contingency in the investment phase. (i.e. incomplete contracting) Instead, firms agree on a list of alternative investment plans that can be carried out in the investment phase, and commit not to consider other alternatives out of the list.[9] Firms may

[8] Although they work with the example of the contract between a pianist and an impresario, the model is of course not limited to the individual level.

[9] In the terminology used by Hart and Moore [2004], they "rule out."

Figure 4.2: Timing of firms' decision.



want to narrow down the list to a single investment plan, but fixing a plan is not usually optimal because initially attractive alternative may turn out to be ex-post suboptimal due to exogenous uncertainty in demand and technology.[10]

Each alternative plan has a different effect on the other firm. Some combinations of firm 1's choice and firm 2's choice create positive synergy while others may create negative one. Using an example of urban renewal, private developer's residential development will benefit from public investments in schools but not from investments in a convention center. I consider a case in which two firms cannot coordinate their decisions to achieve a collectively best outcome and reallocate the aggregate gain. This can be understood as another consequence of incomplete contracting. For example, if a firm realizes synergy in

[10] In the terminology of Hart and Moore [2004], the initial contract does not "rule in."

its particular division, other divisions of the firm may not be able to give away appropriate amount of money to the other firm.

The number of alternatives for firm i, F_i, is used as a measure of flexibility. Keeping more alternatives increases the firm's ability to react to changes in market conditions, but incurs initial cost $L_i(F_i)$. L_i is increasing in F_i at an increasing rate. (i.e. Increasing the number of alternatives from F_i to F_i+1 costs $\Delta L_i(F_i) > 0$, and $\Delta L_i(a) > \Delta L_i(b)$ $for\ a > b$.).[11] Firm i effectively chooses F_i that maximizes the present value of its investment.

In the investment phase, each firm chooses to invest in a particular alternative at a particular time such that it maximizes its firm value. In order to introduce uncertainty in the investment phase, consider a filtered probability space $(\Omega, \mathcal{F}, P)$ with filtration $\{\mathcal{F}_t\}$ on $t \in [0, T]$. t is initialized at zero when the initial agreement is made. Firms must make their investment by the end of prespecified investment phase, T.

Firm i captures value $A_{i,t}^k$ in exchange for its investment in alternative $k \in \{1, 2, ..., F_i\}$ at time t. $A_{i,t}^k$ is fundamental value of firm i's project k that is driven by market conditions. I specify the process $A_{i,t}^k$ as

$$dA_{i,t}^k = A_{i,t}^k \left[\left(r - \mu_i^k \right) dt + \sigma_i^k dW_{i,t}^k \right], \tag{4.0}$$

where r is the risk-free rate, $-\mu_i^k$ is the negative instantaneous drift that represents reduced project value due to early entry of competitors. The joint venture can prevent the reduction in value by starting operation before competitors do and deterring their entries. σ_i^k is volatility of asset value, and $dW_{i,t}^k$ is a one-dimensional standard Wiener process adapted to filtration $\mathcal{F}_t$.

[11] In the example of urban renewal, preparing detailed development plans, updating demand forecast for each type of facility, etc. are more costly as more plans are kept alive.

In addition to the fundamental value, firm i's payoff is influenced by synergy (either positive or negative) that firm j's creates. Firm i is unsure about when firm j changes its investment plans and how large the synergy becomes after the change since firm j's choice is not coordinated with firm i's. I model synergy by a Poisson process $q_{i,t}^k$ such that

$$dq_{i,t}^k = \begin{cases} 0 \text{ with probability } 1 - \lambda dt \\ -\varepsilon_i^k + \tilde{\varepsilon}_i^k \text{ with probability } \lambda dt, \end{cases} \tag{4.0}$$

where ε_i^k is the former level of synergy before jump and $\tilde{\varepsilon}_i^k$ is new level of synergy. The term $-\varepsilon_i^k$ cancels the synergy associated with the former choice of firm j and make the value of synergy stationary around zero. The pdf of $\tilde{\varepsilon}_i^k$, $f(\bullet)$, can be any distribution, but I assume $Uniform(-\sigma_i^s, \sigma_i^s)$ for concreteness.[12]Both λ and σ_i^s are increasing in F_j.since firm j's switching becomes more frequent and variation in combinations increase as F_j becomes larger. I assume $Cov\left(dW_{i,t}^k, dq_i^k\right) = Cov\left(dW_{i,t}^k, \tilde{\varepsilon}_i^k\right) = 0$ for any t, k, i. I now define strategic uncertainty by σ_i^s.

Definition 6 ***Strategic uncertainty*** *is uncertainty in payoff arising from synergy that is brought by the other firm's flexibility. It corresponds to* σ_i^s *in the model.*

A rationale for the strategic uncertainty is provided by the existence of multiple equilibria in the investment phase. A larger strategy space with sustained flexibility results in a wider range of possible equilibria. In the experimental setting of Van Huyck et al. [1991], it is shown that a large strategy space and a large variety of modes of interaction can yield rich dynamics with discrimination among equilibria emerging over time. The theoretical model of Crawford [1995] shows that the probability distribution over limiting

[12]Firm j affects firm i only through strategic uncertainty by letting the expected value of synergy be zero. This is contrasted with the capacity choice model of Appelbaum and Lim [1985] and Vives [1989], where the level effect is studied.

equilibria is generally nondegenerate rather than a particular equilibrium, and that the unique equilibrium cannot be determined solely by the deduction from rationality.

Firm i keeps track of the optimal investment timing and value of each alternative. Let $\Pi_i^k\left(A_{i,t}^k, q_{i,t}^k; K_i^k\right) \equiv \left(A_{i,t}^k + q_{i,t}^k - K_i^k\right)^+$ denote the payoff from immediate investment at time t, where K_i^k is constant investment cost of alternative k. Under no-arbitrage condition, the value of investment opportunity in alternative k as of time t, $c_{i,t}^k$, and the optimal stopping time τ_i^k are, respectively,[13]

$$c_{i,t}^k = ess \sup_{\tau_i^k \in [0,T]} E\left[e^{-r\left(t_i^k - t\right)} \Pi_i^k \left(A_{i,\tau_i^k}^k, q_{i,\tau_i^k}^k; K_i^k \right) \middle| \mathcal{F}_t \right], \tag{4.1}$$

$$\tau_i^k = \inf\left\{ t : c_{i,t}^k = \Pi_i^k \left(A_{i,\tau_i^k}^k, q_{i,\tau_i^k}^k; K_i^k \right) \right\}. \tag{4.2}$$

Firms i keeps track of the best alternative at any given time and make investment when the optimal stopping time arrives. The value of all investment opportunities for firm i is

$$C_{i,t} = \max_i \left\{ c_{i,t}^1, ..., c_{i,t}^{F_i} \right\}. \tag{4.2}$$

Now I derive the value of investment opportunity in alternative k, which is an American call option on an asset whose value is $A_{i,t}^k + q_{i,t}^k$. I suppress superscript k and subscript i ($\bullet_i^k$) for brevity from this point in this section. In the continuation region, the value of investment opportunity can be written as

$$\begin{aligned} c\left(A_t, q_t\right) &= E\left[e^{-rdt} c\left(A_{t+dt}, q_{t+dt}\right) \middle| \mathcal{F}_t \right] \\ &= e^{-rdt} E\left[\begin{array}{c} c_t\left(A_{t+dt}, \varepsilon\right)\left(1 - \lambda dt\right) \\ + c_t\left(A_{t+dt}, \tilde{\varepsilon}\right) \lambda dt \end{array} \middle| \mathcal{F}_t \right]. \end{aligned}$$

[13]Note that agents are risk-neutral.

Applying Ito's lemma, I obtain

$$\begin{aligned}
c(A_t,q_t)\,rdt &= E\left[\begin{array}{c} c(dA_t,\varepsilon)(1-\lambda dt) \\ +\{c(A_t,\tilde{\varepsilon})-c(A_t,\varepsilon)\}\lambda dt \end{array}\middle|\mathcal{F}_t\right] \\
&= E\left[\begin{array}{c} \frac{\partial c_t}{\partial A_t}A_t((r-\mu)dt+\sigma dW_t)+\frac{1}{2}\frac{\partial^2 c_t}{\partial A_t{}^2}(\sigma A_t)^2 dt \\ +\{c(A_t,\tilde{\varepsilon})-c(A_t,\varepsilon)\}\lambda dt \end{array}\middle|\mathcal{F}_t\right] \\
&= \left\{\frac{\partial c_t}{\partial A_t}A_t(r-\mu)+\frac{1}{2}\frac{\partial^2 c_t}{\partial A_t{}^2}(\sigma A_t)^2\right\}dt \\
&\quad +\{E^{\varepsilon}[c(A_t,\tilde{\varepsilon})]-c(A_t,\varepsilon)\}\lambda dt, \qquad (4.\text{-}2)
\end{aligned}$$

where $E^{\varepsilon}[\bullet]$ is expectation over $\tilde{\varepsilon}$. Between time t and $t+dt$, expected return on c_t (in a risk-neutral world) is derived from an expected change in fundamental value (the first term) and an expected change in synergy from ε to $\tilde{\varepsilon}$ (the second term).

In (4.-2), both higher λ and σ_i^s raises $E^{\varepsilon}[c(A_t,q_t)]$ since $c(A_t,q_t)$ is convex in q_t, and thus, they raises the expected change in synergy (the second term). A higher expected change in synergy necessitates the return on c_t to be higher. Since the risk-free rate is exogenously given, c_t needs to be greater. Therefore,

$$\begin{aligned}
\frac{\partial c_t}{\partial \lambda} &> 0, \text{ and} \qquad (4.\text{-}1) \\
\frac{\partial c_t}{\partial \sigma_i^s} &> 0. \qquad (4.0)
\end{aligned}$$

Then the following proposition is derived.

Proposition 7 *Suppose a higher degree of flexibility of firm j (F_j) increases the mean arrival rate (λ) and variance of synergy on firm i (σ_i^s). Then the value of investment opportunity in alternative k for firm i (c_t) is increasing in the degree of flexibility of firm j (F_j).*

Proof. λ and σ_i^s are increasing in F_j. Combined with (4.-1) and (4.0), c_t is also increasing in F_j. ■

By retaining leading terms in dt, I obtain second-order partial differential equation for $c(A_t, q_t)$:

$$0 = -rc_t + \frac{\partial c_t}{\partial A_t} A_t (r - \mu) + \frac{1}{2} \frac{\partial^2 c_t}{\partial A_t{}^2} (\sigma A_t)^2 + \lambda E^{\varepsilon} [c(A_t, \tilde{\varepsilon}, t)] - \lambda c(A_t, \varepsilon, t) . \tag{4.0}$$

If a jump takes into stopping region, $c(A_t, \tilde{\varepsilon})$ is replaced by $\Pi(A_t, \tilde{\varepsilon}; K)$ in (4.4). The problem is a free boundary problem with boundary conditions

$$\begin{aligned} c(A_t, q_T) &= \max\{0, \Pi(A_T, q_T; K)\} \\ c_t(0,0) &= 0 \\ c_\tau(A_\tau, q_\tau) &= \Pi(A_\tau, q_\tau; K) \quad \text{(Value matching condition)} \\ \frac{\partial c_\tau(A_\tau, q_\tau)}{\partial A_\tau} &= 1 \quad \text{(Smooth pasting condition).} \end{aligned}$$

This problem is relatively hard to solve due to the Poisson jump term.

Alternatively, Gukhal [2001] derives a formula for the value of American call option when the underlying asset follows a jump-diffusion process. I can write the value of investment opportunity with the formula:

$$\begin{aligned} c_0 &= b_0 + \int_{u=0}^{T} e^{-ru} E\left[\{\mu A_u - rK\} \mathbf{1}_{\{S_u \in \mathbf{S}\}} \middle| \mathcal{F}_0 \right] du \\ &\quad -\lambda \int_{u=0_+}^{T} e^{-ru} E \left[\begin{array}{c} \{c_u - \Pi(A_u, q_u; K)\} \\ \times \mathbf{1}_{\{S_{u_-} \in \mathbf{S}\}} \mathbf{1}_{\{S_u \in \mathbf{C}\}} \mathbf{1}_{\{u < \tau_j\}} \end{array} \middle| \mathcal{F}_0 \right] du, \end{aligned} \tag{4.-4}$$

where b_0 is the value of corresponding European call option, $\mathbf{1}_{\{\bullet\}}$ is the indicator function, and $S_t \equiv A_t + q_t$. $\mathbf{S}$ and $\mathbf{C} \subset \Re$ represent the stopping and continuation region, respectively.

The critical asset value X_t that divides into $\mathbf{S}$ and $\mathbf{C}$ is obtained by solving $X_t - K = c_t$. $u < \tau_j$ indicates that firm j has not made its investment yet so that the value of synergy may change further.

The sum of two integral terms is called the early exercise premium. The first integral is the benefit from stopped reduction in asset value $(+\mu A_u)$ less the cost of foregone interest $(-rK)$ in cases of early exercise. The second integral is another "cost" of early exercise that is associated with jumps from the stopping region to the continuation region. Even if the option is exercised at time $u_- (= u - du)$ because the asset value is in the stopping region, Poisson jumps may bring the asset value back to the continuation region right after the exercise. In such a case, $c_u - \Pi(A_u, q_u; K) > 0$. The optimal exercise policy requires that the stopped reduction in asset value be greater than the combined cost of foregone interest and a reduced synergy by a jump.

4.5 Analysis

I analyze a reduced-form model based on the detailed model in Section 4 because there is no closed-form solution to the detailed model. I make slight modifications as follows: I assume for ease of analysis that a continuous measure of flexibility, F_i, is defined on $[0, \infty)$. A convex cost function $L_i(F_i)$ is in C^2 such that $L_i(0) = 0$, $L_i'(F_i) > 0$, and $L_i''(F_i) > 0$. Similarly, strategic uncertainty σ_i^s has a derivative with respect to F_j :

$$\frac{d\sigma_i^s}{dF_j} > 0, \ j \neq i. \tag{4.-4}$$

The constant risk-free rate of return is assumed to be zero. Let m denote the index of the chosen investment plan: $m_t = \left\{k \in \{1, ..., F_i\} : c_{i,t}^k \geq c_{i,t}^l \text{ for } \forall l \neq k\right\}$. Then, define

$S_{i,t} \equiv A_{i,t}^m + q_{i,t}^m$ as an envelope of gross payoff process that traces the optimal choice of firm i.

Now I consider properties of $C_{i,t}$, that are characterized by the following equations:

$$C_{i,t} \equiv C_i \left(S_{i,t}, K_i^m, T, t, \sigma_i^m, \sigma_i^s, F_i \right)$$

$$\partial C_{i,t} / \partial S_{i,t} > 0,$$

$$\partial C_{i,t} / \partial K_i^m < 0,$$

$$\partial C_{i,t} / \partial (T - t) > 0,$$

$$\partial C_{i,t} / \partial \sigma_i^s > 0,$$

$$\partial C_{i,t} / \partial F_i \geq 0.$$

Note in particular that (4.-3) and (4.-3) are derived from (4.0) and (4.4), respectively.[14] Furthermore, I make two natural assumptions on the properties of the option value of the investment opportunity. The first assumption is that as firm i increases the degree of flexibility, the marginal benefit of flexibility decreases:

$$\forall F_i, t: \ \partial^2 C_{i,t} / \partial F_i^2 < 0. \tag{4.-3}$$

It is natural that alternatives are examined in the order of their importance. When increasing the degree of flexibility, the firm includes the alternatives that potentially have the greatest impact on the value. The second assumption is that the marginal benefit of flexibility is greater when there is more uncertainty:

$$\forall F_i, t: \ \partial^2 C_{i,t} / \partial F_i \partial \sigma_i^m > 0, \text{ and } \partial^2 C_{i,t} / \partial F_i \partial \sigma_i^s > 0. \tag{4.-3}$$

[14] Triantis and Hodder [1990] derive the positive option value of flexibility in production system.

The more uncertain the economy is, the more valuable the state contingency becomes.

Going back to the initial phase of negotiation, each firm's problem is to maximize the initial value of its project $V_i \equiv C_{i,0} - L_i(F_i)$ over F_i:

$$\max_{F_i} V_i.$$

Note that $V_i(F_i)$ is concave in F_i from (4.5) and $L_i''(F_i) > 0$. Suppose for now that σ_i^s is independent of F_i. By the concavity of V_i, the necessary and sufficient condition that characterizes the optimal F_i^* for $i = \{1, 2\}$is

$$\frac{\partial V_i}{\partial F_i} = -\frac{\partial L_i}{\partial F_i} + \frac{\partial C_{i,0}}{\partial F_i} = 0. \tag{4.-3}$$

Let $F_i(\sigma_i^s(F_j); S_{i,0}, K_i^m, T)$ denote the reaction function of firm i which is implicitly defined by (4.5).

Next, I show that σ_i^s is affected by its own choice of flexibility; what I call the strategic uncertainty effect, $\frac{\partial \sigma_i^s}{\partial F_i}$, is indeed positive. Strategic complementarity is a key to understanding it.[15]

Lemma 8 *When there is a strategic uncertainty, F_i and F_j are strategic complements for $i = 1, 2$. That is, a more flexible strategy of firm i results in the firm j's reaction with a more flexible strategy:*

$$\left.\frac{\partial F_j(F_i)}{\partial F_i}\right|_{\frac{\partial V_j}{\partial F_j}=0} > 0. \tag{4.-3}$$

Proof. See Appendix E. ■

Now I define the strategic uncertainty effect (i.e. the effect of flexibility on own uncertainty).

[15] This terminology is derived from Bulow et al. [1985]. A textbook treatment is provided by Varian [1992].

Definition 9 *The* ***strategic uncertainty effect*** *is the effect of a firm's choice of flexibility on the uncertainty that the firm itself faces* ($\frac{\partial \sigma_i^s}{\partial F_i}$)*, when there is strategic uncertainty.*

Then, the following proposition is readily derived.

Proposition 10 *The strategic uncertainty effect is positive. That is, increased (decreased) degree of flexibility by a firm increases (decreases) the uncertainty that the firm itself faces.*

$$\frac{\partial \sigma_i^s}{\partial F_i} > 0, \ i = 1, 2 \tag{4.-3}$$

.

Proof. From (4.5), σ_i^s is strictly increasing in F_j (i.e. $\partial \sigma_i^s / \partial F_j > 0$) . From (8), $\partial F_j / \partial F_i$ is strictly positive at the optimum. Then, $\frac{\partial \sigma_i^s}{\partial F_i} = \frac{\partial \sigma_i^s}{\partial F_j} \frac{\partial F_j}{\partial F_i} > 0$ is readily derived. ■

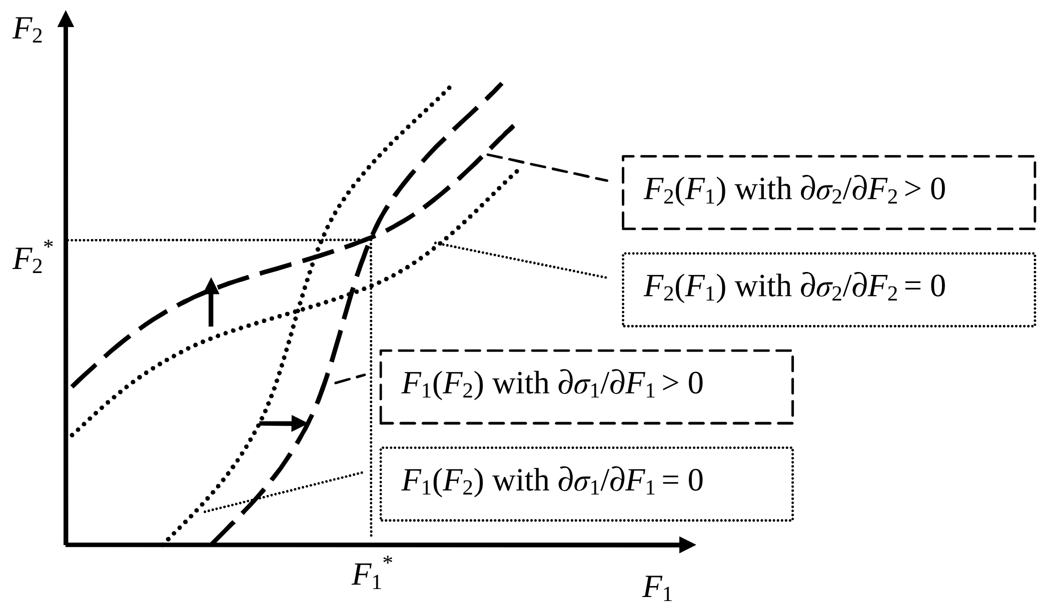
With the strategic uncertainty effect, the optimality condition for $i = \{1, 2\}$ becomes

$$\frac{\partial V_i}{\partial F_i} = -\frac{\partial L_i}{\partial F_i} + \frac{\partial C_{i,0}}{\partial F_i} + \frac{\partial C_{i,0}}{\partial \sigma_i^s} \frac{\partial \sigma_i^s}{\partial F_i} = 0 \tag{4.-3}$$

The introduction of the strategic uncertainty will shift the reaction function outward; that is, the reaction function is everywhere higher in (4.5) than in (4.5). To see this, note that the additional term $\frac{\partial C_{i,0}}{\partial \sigma_i^s} \frac{\partial \sigma_i^s}{\partial F_i}$ is positive by (4.-3) and (10). Then $\frac{dC_{i,0}}{dF_i} \left(= \frac{\partial C_{i,0}}{\partial F_i} + \frac{\partial C_{i,0}}{\partial \sigma_i^s} \frac{\partial \sigma_i^s}{\partial F_i} \right)$ is higher for any level of F_i since F_i has now two channels that positively affect the value of investment option in (4.5). Second, $L_i^{'}(F_i)$is increasing in F_i. Thus, from the same arguments as in the proof of lemma, the optimal level of flexibility for firm i is higher for any given level of F_j. (See Figure E.1.) Now I obtain the following proposition.

Proposition 11 *The equilibrium degree of flexibility* F_i^* *for* $i = 1, 2$ *is higher when the strategic uncertainty effect exists.*

Figure 4.3: Reaction functions.



$F_i\,(F_j)$ is the reaction function of firm i to the strategy of firm j. Strategic complementarity implies an upward sloping reaction function. With a positive strategic uncertainty effect ($\partial\sigma_i/\partial F_i > 0$), the reaction function is everywhere higher and, as a result, the equilibrium degree of flexibility F_i^* is higher.

Proof. With the strategic uncertainty effect, each firm must satisfy (4.5) at the optimum. Then the reaction curve of each firm is shifted outward as explained in the discussion above. Since the strategic complementarity (8) implies upward sloping reaction curves, an outward shift of a reaction curve results in a higher equilibrium level of flexibility for each firm. ■

A symmetric case is depicted in Figure 4.3. Non-decreasing reaction functions represent strategic complementarity in (8).

Note that there must be at least one equilibrium level of flexibility, provided that the agreement is reached in the initial phase. Although the uniqueness cannot be proved without specifying the functional form of the reaction curves, the level of flexibility in the contract either represents unique equilibrium or is uniquely determined among multiple equilibria through negotiation process (e.g. Nash bargaining) that is not modeled in this chapter.

Now let's consider the equilibrium timing of investment. After deciding the degree of flexibility of the investment plan, the firms choose the optimal timing of investment as seen in (4.2). At each point in time until the expiration of investment option, the firms make decisions based on the optimal investment rule:

$$\forall t \in (0, T] : \text{ Invest if } I_{i,t} \equiv S_{i,t} - K_i^m - C_{i,t} \geq 0. \tag{4.-5}$$

Note that the Tobin's q criteria under perfect competition is $S_{i,t} - K_i^m \geq 0$. Here, the criteria takes account of the opportunity cost of discarding options when the firm makes investment. Incorporating Proposition 6 into this investment rule, I obtain the following proposition about the equilibrium investment timing.

Proposition 12 *Investment is more delayed with a positive strategic uncertainty effect* $\frac{\partial \sigma_i^s}{\partial F_i} > 0$.

Proof. Since the equilibrium degree of flexibility is higher when there is strategic uncertainty effect (Proposition 6), the resulting level of uncertainty is higher for both firms.

Then, the equilibrium value of investment opportunity $C^*_{i,t}$ is higher for all t for two reasons. First, a higher equilibrium level of flexibility weakly raises the value of investment opportunity through the direct effect of (4.-3). Second, a higher equilibrium level of flexibility increases uncertainty through the strategic uncertainty effect (10), and then the increased uncertainty raises the value of the investment opportunity through (4.-3). In order for the investment to become optimal, $S_{i,t}$ needs to be higher. As a result, the investment is more delayed with the strategic uncertainty effect. ■

4.6 Concluding Remarks

We often face the situation where too little constraint, or too much freedom, makes decision-making harder due to enormous number of possible scenarios. Decisions are also postponed to wait for the uncertainty to be resolved.

Corporate investment decisions are often made under such environments. The existing studies consider the effects of exogenous uncertainty on the investment decision and on the choice of flexibility. It is shown that agents prefer to have more flexibility when faced with uncertainty. The innovation of this chapter is to incorporate the reverse effect: The effect of flexibility on uncertainty. Strategic uncertainty is created by the choice of a higher degree of flexibility. I show feed-back effects of strategic uncertainty to the choice of flexibility, and to the investment-timing decision. With the strategic uncertainty effect, the equilibrium degree of flexibility is higher and the investment is more likely to be delayed. This self-reinforcing mechanism is important since the optimal decision-making of individual agents results in a stagnant economic activities.

The mechanism applies not only to the investment decision but also to a variety of negotiations. When manufacturers are trying to decide a new product standard, such as video format and DVD format, there is significant uncertainty and the firms often hesitate to commit to any of the proposed format. Uncertainty stems from the strategic flexibility that the other parties keep. Another example is the multinational negotiation in the international organizations when forming a joint multinational project. The model of the strategic uncertainty effect sheds light on these wide range of strategic activities.

The current model can be extended in many directions. Future works may include 1) to derive more detailed characterization of the link between the other firm's flexibility and the strategic uncertainty, and 2) to test empirically the null hypotheses derived from the model. In particular, it would be interesting to compare the expected time to complete a project by joint ventures with that by single firms, after controlling for other characteristics such as scale and product.

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Appendix A

Derivation of the equilibrium in Chapter 2

In this appendix, I describe how to solve for the equilibrium that is defined in Chapter 2.

(Labor markets) Labor supply is $L_t^{\text{sup}} = 1$. Labor demand is derived from the first order condition of a goods-producing firm (2.6b): $w_t = (1-\alpha) A_t (K_t/L_t)^{\alpha}$. Using the capital demand from another first order condition (2.6a), the equilibrium wage is derived as a function of A_t and i_t :

$$w_t^{eq}(A_t, i_t) = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} (i_t - 1 + \delta)^{-\frac{\alpha}{1-\alpha}}.$$

(Land markets) Land supply is $T_t = r_t^{\mu}$. Land demand is derived from the first order condition of a real estate firm (2.7b): $T_t^{dem} = \{(1-\gamma) B_t p_t / r_t\}^{\frac{1}{\gamma}} S_t$. Using demand for housing structures from another first order condition (2.7a), the equilibrium land rent and the quantity of land is derived as a function of B_t, p_t, i_t :

$$\begin{aligned} r_t^{eq}(B_t, p_t, i_t) &= \gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma) B_t^{\frac{1}{1-\gamma}} p_t^{\frac{1}{1-\gamma}} (i_t - 1 + \delta)^{-\frac{\gamma}{1-\gamma}}, \\ T_t^{eq}(B_t, p_t, i_t) &= \gamma^{\frac{\gamma\mu}{1-\gamma}}(1-\gamma)^{\mu} B_t^{\frac{\mu}{1-\gamma}} p_t^{\frac{\mu}{1-\gamma}} (i_t - 1 + \delta)^{-\frac{\gamma\mu}{1-\gamma}}. \end{aligned}$$

Although both r_t and T_t depend on the housing rent (p_t), r_t and T_t can be written as functions of B_t, A_1, A_2, i_1, i_2 after deriving the equilibria of the other markets.

(Housing markets) Housing supply is $H_t^{\text{sup}}(p_t; B_t, i_t) = B_t S_t^{eq}(B_t, p_t, i_t)^{\gamma} \times T_t^{eq}(B_t, p_t, i_t)^{1-\gamma}$. Housing demand is derived as (2.9c) from the first-order conditions of the households. Analytical solution to the housing market equilibrium is available for the log case:

$$\begin{aligned} p_1^{eq}(i_1, Inc) &= \{2(1+\beta)\}^{-\frac{1-\gamma}{1+\mu}} \gamma^{-\gamma} (1-\gamma)^{-\frac{\mu(1-\gamma)}{1+\mu}} B_1^{-1} (i_1 - 1 + \delta)^{\gamma} Inc^{\frac{1-\gamma}{1+\mu}}, \\ H_1^{eq}(i_1, Inc) &= \{2(1+\beta)\}^{-\frac{\gamma+\mu}{1+\mu}} \gamma^{\gamma} (1-\gamma)^{\frac{\mu(1-\gamma)}{1+\mu}} B_1 (i_1 - 1 + \delta)^{-\gamma} Inc^{\frac{\gamma+\mu}{1+\mu}}. \end{aligned}$$

For the CES-CRRA case, a numerical solution must be used to derive p_1, p_2, p_1^*, p_2^* jointly with i_1 an i_2.

(Capital markets) After obtaining $p_1^{eq}(i_1, Inc)$ and $H_1^{eq}(i_1, Inc)$ for the log case, I can rewrite $r_t^{eq}(i_t, Inc)$ and $T_t^{eq}(i_t, Inc)$.and further derive Inc as

$$\begin{aligned} Inc(A_1, A_2, i_1, i_2) &= i_1 W_0 + r_1 T_1(i_1, Inc) + w_1(A_1, i_1) \\ &\quad + \frac{1}{i_2}\{r_2 T_2(i_2, Inc) + w_2(A_2, i_2)\} \\ &= 2(1+\gamma)^{-1} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) \\ &\quad \times \left\{ A_1^{\frac{1}{1-\alpha}} (i_1 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} + A_2^{\frac{1}{1-\alpha}} i_2^{-1} (i_2 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} \right\}. \end{aligned}$$

Note that B_t does not appear in land rents or land quantity in the log-utility case while it does appear in the CES-CRRA case.

Now the capital supply for period 2, W_1, is derived. Given $Inc\,(A_1, A_2, i_1, i_2)$, the consumption becomes $C_t\,(A_1, A_2, i_1, i_2)$ and the households' saving after period 1 is

$$\begin{aligned} W_1\,(A_1, A_2, i_1, i_2) &= i_1 W_0 + r_1 T_1\,(A_1, A_2, i_1, i_2) + w_1\,(A_1, i_1) \\ &\quad - C_1\,(A_1, A_2, i_1, i_2) - p_1 H_1\,(A_1, A_2, i_1, i_2)\,. \end{aligned}$$

The market-clearing conditions in capital markets are

$$W_0 + W_0^* = \begin{bmatrix} K_1\,(A_1, i_1) + K_1^*\,(A_1^*, i_1) \\ +S_1\,(A_1, A_2, i_1, i_2) + S_1^*\,(A_1^*, A_2^*, i_1, i_2) \end{bmatrix} \text{ (for t=1)},$$

$$\begin{bmatrix} W_1\,(A_1, A_2, i_1, i_2, W_0) \\ +W_1^*\,(A_1^*, A_2^*, i_1, i_2, W_0) \end{bmatrix} = \begin{bmatrix} K_2\,(A_2, i_2) + K_2^*\,(A_2^*, i_2) \\ +S_2\,(A_1, A_2, i_1, i_2) + S_2^*\,(A_1^*, A_2^*, i_1, i_2) \end{bmatrix} \text{ (for t=2)}.$$

With these two equations, in principle two unknowns (i_1, i_2) can be solved for in terms of the exogenous variables $(A_1, A_2, A_1^*, A_2^*, W_0, W_0^*)$. Numerical solutions must be used to obtain the actual solutions.

In the case of CES-CRRA, capital markets' equilibria will depend additionally on the housing rents. Therefore, the housing-market equilibrium and the capital-market equilibrium are solved simultaneously.

In this chapter, basic parameters are set as follows: $\alpha = 1/3, \beta = 0.9, \gamma = 0.7, \delta = 0.5$.

Appendix B

Derivation of the moment conditions in Chapter 3

In this appendix, we derive the intra-temporal and inter-temporal optimality conditions for a dynamic consumption problem of the representative consumer with the CES utility function. The planner's dynamic programming consists of the following Bellman equation and the accumulation equation.

$$\begin{aligned} V_t\left(S_{t-1}\right) &= \Omega\left[u\left(C_t, H_t\right)\right] + \beta E_t\left[V_{t+1}\left(S_t\right)\right] \\ s.t. \quad S_t &= Y_t + S_{t-1}\left(\mathbf{q}_{t-1}'\mathbf{R}_t\right) - C_t - P_t H_t, \end{aligned}$$

The first-order conditions with respect to C and H are

$$\Omega_{C,t} = \beta E_t\left[V_{t+1}'\left(S_t\right)\right], \tag{B.-1}$$

$$\Omega_{H,t} = \beta E_t\left[V_{t+1}'\left(S_t\right)P_t\right], \tag{B.0}$$

where $\Omega_{C,t} \equiv \partial\Omega\left(u\left(C_t, H_t\right)\right)/\partial C_t, \Omega_{H,t} \equiv \partial\Omega\left(u\left(C_t, H_t\right)\right)/\partial H_t$.

From these equations, we obtain an intra-temporal optimality condition (3.-3) that the price of housing services is equal to the absolute value of marginal rate of substitution between two goods:

$$P_t = \frac{\Omega_{H,t}}{\Omega_{C,t}} = \frac{\alpha}{1-\alpha}\left(\frac{H_t}{C_t}\right)^{-\frac{1}{\rho}}.$$

Applying the envelope condition to the Bellman equation, the Benveniste-Sheinkman condition is

$$V_t'\left(S_{t-1}\right) = \beta E_t\left[V_{t+1}'\left(S_t\right)\right]\left(\mathbf{q}_{t-1}'\mathbf{R}_t\right).$$

Combining with (B.-1) and shifting one period forward, we obtain

$$V_{t+1}'\left(S_t\right) = \Omega_{C,t+1}\left(\mathbf{q}_t'\mathbf{R}_{t+1}\right).$$

By plugging in (B.-1), we obtain the Euler equation with respect to the portfolio return:

$$1 = E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}\left(\mathbf{q}_t'\mathbf{R}_{t+1}\right)\right].$$

The pricing kernel $\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}$ is

$$\begin{aligned}
\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}} &= \beta\left\{\left(\frac{C_{t+1}}{C_t}\right)\left[\frac{1-\alpha+\alpha\left(\frac{H_{t+1}^{\eta}}{C_{t+1}}\right)^{1-\frac{1}{\rho}}}{1-\alpha+\alpha\left(\frac{H_t^{\eta}}{C_t}\right)^{1-\frac{1}{\rho}}}\right]^{\frac{\theta-\rho}{1-\rho}}\right\}^{-\frac{1}{\theta}} \\
&= \beta\left\{\left(\frac{C_{t+1}}{C_t}\right)\left[\frac{\left[(1-\alpha)^{\rho}+\alpha^{\rho}\left(\frac{\rho-1}{\rho-\eta}P_{t+1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[(1-\alpha)^{\rho}+\alpha^{\rho}\left(\frac{\rho-1}{\rho-\eta}P_t\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}\right]^{\theta-\rho}\right\}^{-\frac{1}{\theta}}.
\end{aligned}$$

The optimal portfolio choice can be reformulated as

$$\forall t: \quad \max_{\mathbf{q}_t} E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}\left(\mathbf{q}_t'\mathbf{R}_{t+1}\right)\right] \qquad s.t..\ \mathbf{q}_t'\iota = 1.$$

The first-order condition for the ith asset is

$$E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}R^i_{t+1}\right]=\lambda_t,$$

where λ_t is the Lagrange multiplier. Equating the above equation for arbitrary assets i and j, we obtain the Euler equation for excess return:

$$E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}\left(R^i_{t+1}-R^j_{t+1}\right)\right]=0.$$

The expected value of any excess return weighted by the pricing kernel must be zero. By multiplying both sides by q^i_t and summing all N assets,

$$\begin{aligned} 0 &= E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}\left(\sum_{i=1}^{N}q^i_t\left(R^i_{t+1}-R^j_{t+1}\right)\right)\right] \\ &= E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}\left(\mathbf{q}'_t\mathbf{R}_{t+1}-R^j_{t+1}\right)\right]. \end{aligned}$$

Recalling the Euler equation for the portfolio return, we obtain the inter-temporal optimality condition (3.-2) for any asset j,

$$E_t\left[\beta\frac{\Omega_{C,t+1}}{\Omega_{C,t}}R^j_{t+1}\right]=1.$$

Appendix C

Relationship between income expansion and parameter η in Chapter 3

In this appendix, we show that a low value of η in Pakos' felicity function indicates a positive relationship between consumption share of housing and income (i.e. $sgn\left(\partial\left(H/C\right)/\partial Y\right) = sgn\left(1-\eta\right)$).

By solving the static problem (3.3.1) using Pakos' felicity function (3.2.1), we obtain the following equations that implicitly define demands for housing and non-housing.

$$\begin{aligned} PH + \left(\frac{1-\alpha}{\alpha}\frac{\rho-1}{\rho-\eta}P\right)^{\rho} H^{\eta} &= Y, \\ C + \left(\frac{1-\alpha}{\alpha}\frac{\rho-1}{\rho-\eta}\right)^{-\rho/\eta} P^{1-\rho/\eta} C^{1/\eta} &= Y. \end{aligned}$$

If $\eta = 1$, demand functions reduce to the CES case. We want income elasticity of consumption ratio:

$$\begin{aligned}\varepsilon_Y^{H/C} &\equiv \frac{\partial \ln (H/C)}{\partial \ln Y} \\ &= \frac{\partial \ln H}{\partial \ln Y} - \frac{\partial \ln C}{\partial \ln Y} \\ &= \varepsilon_Y^H - \varepsilon_Y^C.\end{aligned}$$

Income elasticities of housing demand (ε_Y^H) and non-housing demand (ε_Y^C) are, respectively,

$$\begin{aligned}\varepsilon_Y^H &\equiv \frac{\partial H}{\partial Y}\frac{Y}{H} \\ &= \frac{1}{P + \left(\frac{1-\alpha}{\alpha}\frac{\rho-1}{\rho-\eta}P\right)^{\rho} H^{\eta-1}\eta}\frac{Y}{H} \\ &= \frac{Y/H}{P + (Y/H - P)\,\eta} \\ &= \left[\frac{PH}{Y} + \eta\frac{C}{Y}\right]^{-1}, \\ \varepsilon_Y^C &\equiv \frac{\partial C}{\partial Y}\frac{Y}{C} \\ &= \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\frac{\rho-1}{\rho-\eta}\right)^{-\rho/\eta} P^{1-\rho/\eta} C^{1/\eta-1}\eta}\frac{Y}{C} \\ &= \frac{Y/C}{1 + (Y/C - 1)\,/\eta} \\ &= \left[\frac{1}{\eta}\frac{PH}{Y} + \frac{C}{Y}\right]^{-1}.\end{aligned}$$

Therefore, income elasticity of consumption ratio is

$$\begin{aligned}\varepsilon_Y^{H/C} &= \left[\frac{PH}{Y} + \eta\frac{C}{Y}\right]^{-1} - \left[\frac{1}{\eta}\frac{PH}{Y} + \frac{C}{Y}\right]^{-1} \\ &= \left[\frac{1}{\eta}\frac{PH}{Y} + \frac{C}{Y} - \frac{PH}{Y} - \eta\frac{C}{Y}\right]\left[\left(\frac{PH}{Y} + \eta\frac{C}{Y}\right)\left(\frac{1}{\eta}\frac{PH}{Y} + \frac{C}{Y}\right)\right]^{-1} \\ &= (1-\eta)\left(\frac{PH}{Y} + \eta\frac{C}{Y}\right)^{-1}.\end{aligned}$$

The sign of income elasticity of consumption ratio is equal to the sign of $1 - \eta$: If η is less than one, ceteris paribus, a higher income is associated with a greater share of housing consumption.

Appendix D

Heterogeneous households with two types in Chapter 3

We examine the following formulation of heterogeneous consumers by slightly modifying the representative consumer with the CES-power utility. The economy now consists of two consumers, one with labor income only (labeled as employee) and the other with financial income only (labeled as investor). Each consumer maximizes CES utility. By estimating the relationship between consumption share of housing and income share of employee, we can infer relative levels of substitution parameters for different types of consumers.

Augmenting the CES-utility function, we allow for heterogeneous consumers, with $k = \{e, i\}$ representing employee and investor, respectively. The static problem (3.3.1) becomes

$$\max_{C^k,H^k} u\left(C^k, H^k\right)$$

$$s.t.\ \ C^k + PH^k = w^k Y \quad for\ k = \{e, i\},$$

where w^k is income share of consumer k such that $w^e + w^i = 1$. The utility function is modified as

$$u\left(C^k, H^k\right) = \left[\left(1-\alpha^k\right) C^{k\,1-1/\rho^k} + \alpha^k\ H^{k\,1-1/\rho^k}\right]^{1/\left(1-1/\rho^k\right)}$$

so that each consumer exhibits homothetic preferences. The demand functions of consumer k for housing services and non-housing good are

$$H^k = w^k Y \left[P + \left(\frac{1-\alpha^k}{\alpha^k} P\right)^{\rho^k}\right]^{-1}, \quad \text{(D.1)}$$

$$C^k = w^k Y \left[1 + \left(\frac{\alpha^k}{1-\alpha^k}\right)^{\rho^k} P^{1-\rho^k}\right]^{-1}. \quad \text{(D.2)}$$

Total demand for housing services and non-housing goods are $H = H^e + H^i$ and $C = C^e + C^i$, respectively. In this economy, income share of investor (w^i) affects consumption ratio of housing to non-housing while total wealth level (Y) has no effect. The effect of income share on consumption ratio when price is kept constant is obtained by taking partial derivative of H/C with respect to w^i.

$$\frac{\partial\left(H/C\right)}{\partial w^i} = \frac{\left(A^e - A^i\right)}{\left(A^e+P\right)\left(A^i+P\right)} \left[\frac{A^e}{A^e+P} + w^i\left(\frac{A^i}{A^i+P} - \frac{A^e}{A^e+P}\right)\right]^{-2}$$

where $A^k \equiv \left(P\left(1-\alpha^k\right)/\alpha^k\right)^{\rho^k}$. The sign of the partial derivative is determined by relative size of A^k:

$$sgn\left[\frac{\partial\left(H/C\right)}{\partial w^i}\right] = sgn\left(A^e - A^i\right).$$

One of the simplest cases is $P\left(1-\alpha^{k}\right)/\alpha^{k} > 1$ for $k = e, i$. Then the sign of coefficient is determined by SES ρ: $sgn\left(\partial\left(H/C\right)/\partial w^{i}\right) = sgn\left(\rho^{e} - \rho^{i}\right)$: A positive effect of income share of investor on consumption ratio of housing to non-housing $\left(\partial\left(H/C\right)/\partial w^{i} > 0\right)$ implies that investor has a lower elasticity of substitution between housing services and non-housing goods than employee does $\left(\rho^{i} < \rho^{e}\right)$, and vice versa.

Appendix E

Proof of Lemma in Chapter 4

Proof of Lemma. Firm j's optimal degree of flexibility is determined where $\frac{\partial L_j}{\partial F_j} = \frac{\partial C_{j,0}}{\partial F_j}$ from the optimality condition (4.5). Note that $\frac{\partial C_{j,0}}{\partial F_j}$ is decreasing in F_j from (4.5) and $\frac{\partial L_j}{\partial F_j}$ is increasing in F_j by assumption. (See Figure E.1.) Now suppose that firm i increases its degree of flexibility F_i to have more choices of action. Then, firm j faces a higher uncertainty due to the existence of strategic uncertainty ($\partial \sigma_j^S / \partial F_i > 0$ (4.5)). By (4.5), $\frac{\partial C_{j,0}}{\partial F_j}$ is higher for any level of F_j when σ_j^S is large.[1] This shifts $\frac{\partial C_{j,0}}{\partial F_j}$ upward in Figure E.1. Since $L_j'(F_j)$ is upward sloping, the optimal response of firm j is to choose a strictly greater F_j. That is, greater uncertainty leads to an adaptive choice of greater flexibility. Therefore, a more flexible strategy of firm i results in the firm j's reaction with a more flexible strategy:

$$\left.\frac{\partial F_j(F_i)}{\partial F_i}\right|_{\frac{\partial V_j}{\partial F_j}=0} > 0. \tag{E.0}$$

[1] Even without the assumption (4.5), we can still argue that the F_j is more likely to be higher when σ_j^S is greater. When σ_j^S is high, the option to invest for firm j has a higher value $\left(\partial C_{j,0}(\sigma_j^S, F_j)/\partial \sigma_j^S > 0\right)$ for any given level of F_j. Then it is more likely that the condition on the first line of (4.5) is met. Therefore, there is a higher probability to have positive F_j.

It is easy to show the equivalence of (8) to the standard definition of strategic complementarity:

$$\frac{\partial^2 V_j}{\partial F_i \partial F_j} > 0. \tag{E.0}$$

Substituting the reaction function $F_i\left(\sigma_i^S\left(F_j\right); S_{i,0}, K_i^m, T\right)$ into the first line of (4.5) ($\frac{\partial Vj}{\partial Fj} = 0$) and differentiating,

$$\frac{\partial^2 V_j\left(F_j\left(F_i\right), F_i\right)}{\partial F_j^2}\frac{\partial F_j\left(F_i\right)}{\partial F_i} + \frac{\partial^2 V_j\left(F_j\left(F_i\right), F_i\right)}{\partial F_i \partial F_j} = 0.$$

Rearranging gives

$$\left.\frac{\partial F_j\left(F_i\right)}{\partial F_i}\right|_{\frac{\partial V_j}{\partial F_j}=0} = -\left.\frac{\partial^2 V_j\left(F_j\left(F_i\right), F_i\right)}{\partial F_i \partial F_j}\right/\frac{\partial^2 V_j\left(F_j\left(F_i\right), F_i\right)}{\partial F_j^2}.$$

Since $V_j\left(F_j\right)$ is concave by assumption,

$$sgn\left(\left.\frac{\partial F_j\left(F_i\right)}{\partial F_i}\right|_{\frac{\partial V_j}{\partial F_j}=0}\right) = sgn\left(\frac{\partial^2 V_j\left(F_j\left(F_i\right), F_i\right)}{\partial F_i \partial F_j}\right). \tag{E.0}$$

Therefore, $\left.\frac{\partial F_j(F_i)}{\partial F_i}\right|_{\frac{\partial V_j}{\partial F_j}=0} > 0$ is equivalent to $\frac{\partial^2 V^i}{\partial F_i \partial F_j} > 0$. ■

Figure E.1: Effect of volatility on flexibility.