A Simple Hybrid Model for Estimating Real Estate Price Indexes

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The accurate measurement of housing and real estate prices is of real theoretical importance and is crucial to understanding the operation of the housing market. Two basic techniques—hedonic and repeat sales methods—for measuring and analyzing the structure of housing prices have been developed. This paper presents an explicit model for combining samples of single sales and multiple sales for the analysis of housing prices and the computation of more efficient price indices. The procedure is based upon an explicit error structure, incorporating a random walk in housing prices. It is also based upon robust generalized least-squares methods to improve the efficiency of estimation. The technique is illustrated using a unique sample of condominium sales during a 12-year period in downtown Los Angeles. © 1995 Academic Press, Inc.

1. INTRODUCTION

The accurate measurement of housing and real estate price trends is of real theoretical importance and is crucial to understanding the operation of the housing market. For example, recent conclusions about the efficiency of the housing market (Case and Shiller, 1989) depend upon price indexes to measure the returns to arbitrage. Models of consumer behavior, and in particular models which investigate the determinants of speculative price bubbles in real estate (Abraham and Hendershott, 1994), depend upon the accurate measurement of those prices.

The accurate measurement of housing prices is also of enormous practical importance. Applications range from the estimation of regional variation in the cost of living to the computation of transfer payments to the

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indigent. More recently, as primary housing markets have become more integrated with secondary markets, the computation of housing prices has become of great practical importance to investors who confront choices among portfolios composed of housing securities and other investment assets (Shiller, 1993).

Two basic techniques for measuring and analyzing the structure of housing prices have been developed. For more than 20 years, hedonic models (Kain and Quigley, 1970) have been used to analyze market prices; repeat sales models have been utilized for an even longer period of time (Bailey et al., 1963).

Hedonic models relate the selling prices (or monthly rents) of dwellings to measures of their physical and locational characteristics and to some representation of time. Hedonic models are routinely estimated from repeated cross-sectional samples of dwellings, for example, housing prices and characteristics reported by multiple listing services, or local government agencies or in surveys (see, for example, Pollakowski (1987)). However, neither the functional form of the relationship nor the set of variables is known with certainty; this limits the generality of the procedure when applied across markets or time periods.

Repeat sales models avoid these difficulties by measuring the price of the same house at several points in time. This obviates the need to measure the characteristics of houses (but only as long as these characteristics have remained constant between sales). It also limits the samples available for analysis. More important than the smaller size of repeat sales samples is the possibility that they may not be representative of local housing markets, at least not in the short or medium run (Gatzlaff and Haurin, 1993).

Repeat sales models typically specify a random walk in housing prices (Case and Shiller, 1987; Abraham and Schauman, 1991; Quigley and Van Order, in press).

This paper presents an explicit procedure for combining samples of single sales and multiple sales in the analysis of housing prices and in the computation of price indices. This is not the first proposal for combining information across samples within a single market. Indeed, the so-called hybrid model (Case and Quigley, 1991) has been used extensively in recent research (Case et al., 1991). However, in contrast to previous work, the procedure introduced in this paper is based upon an explicit error structure.¹

¹ After this paper was drafted, we became aware of other recent work (Hill et al., 1994) which combines single sales and multiple sales using an explicit error structure, but relying upon maximum likelihood techniques. In contrast, the procedure discussed below relies upon robust generalized least squares models to achieve asymptotic efficiency. The procedure introduced below also follows most of the recent literature in specifying a random walk in housing prices.
Section II below presents the basic model. Section III presents an empirical analysis comparing the estimation of house price indexes. This empirical analysis is based upon an unusual and quite special sample of condominium dwellings in the downtown Los Angeles area. The empirical analysis reported below is based upon essentially all sales of condominium apartments in the downtown Los Angeles area during a 12-year period. The sample is unusually well suited to the comparative analysis. The dwellings in the sample are all drawn from a few high-rise buildings located within a quarter mile of each other. Thus their locational and public service attributes are quite similar. The sample also includes a large number of repeat sales, facilitating a comparison of alternative estimators of price indices.

II. THE MODEL

The sale value or monthly rent of a housing unit represents an amalgam, \(PQ\), of an index representing the price, \(P\), of housing and another representing the level of services, \(Q\), emitted by that unit. To represent this, suppose

\[
V_{at} = Q_{at} + P_t + \omega_{at}. \tag{1}
\]

where \(V_{at}\) is the logarithm of the observed selling price of house \(i\) at time \(t\), \(Q_{at}\) is the log of the quality of house \(i\) sold at time \(t\), and \(P_t\) is the log of the constant quality housing price index at time \(t\). \(\omega_{at}\) is a random error.

According to Eq. (1), each house emits a quality of service \(Q_{at}\) which is priced at \(P_t\) at a particular point in time. \(Q_{at}\) is unobserved, but

\[
Q_{at} = \beta X_{at} + \xi_t + \eta_{at}. \tag{2}
\]

According to Eq. (2), housing quality is a function of a vector of observable characteristics of dwellings at time \(t\), \(X_{at}\), and a dwelling-unit-specific factor, \(\xi_t\). The term \(\xi_t\) represents the unmeasured characteristics of house \(i\), and \(\eta_{at}\) is a random error. Combining (1) and (2) yields

\[
V_{at} = \beta X_{at} + P_t + \delta_{at}. \tag{3}
\]

where \(\delta_{at}\) is the composite error term,

\[
\delta_{at} = \xi_t + \eta_{at} + \omega_{at} = \xi_t + \epsilon_{it}. \tag{4}
\]
Assume
\[ E(\xi_i) = 0 \quad E(e_{it}) = 0 \]
\[ E(\xi_i^2) = \sigma^2_\xi \quad E(e_{it}^2) = \sigma^2_e \]
\[ E(\xi_i \xi_j) = 0 \quad E(e_{it} e_{jt}) = 0. \]  \hspace{1cm} (5)
\[ E(\xi_i e_{it}) = 0 \]

Suppose, based on empirical evidence,\(^2\) that housing prices follow a random walk such that
\[ E(e_{it} - e_{ir})^2/\sigma^2_e = A(t - \tau) + B(t - \tau)^2. \]  \hspace{1cm} (6)

According to Eq. (6), if \( A \) is positive, the variance in housing price for an individual dwelling increases with the elapsed time between its first sale, \( \tau \), and a subsequent sale at time \( t \). If \( B \) is negative, then the variance increases at a decreasing rate.

If all dwellings in a given sample are repeat sales, all the parameters of the model can be estimated in an asymptotically efficient manner.

Multiple sales provide two important sources of information in estimating the model. First, they permit the systematic components of housing quality to be distinguished from the idiosyncratic components that vary among individual dwelling units. Second, they permit the variance–covariance matrix of disturbances to be estimated, thus improving the efficiency of estimation of the price index \( P_t \) as well as the parameters \( \beta \).

A sample of single sales permits Eq. (3) to be estimated, but it does not permit the measured characteristics of houses to be distinguished from the unmeasured, individual-specific, characteristics of those dwelling units. Presumably, many characteristics of individual houses that are difficult to measure quantitatively, particularly in a large sample, are important in affecting house values. Samples of individual sales do not permit these idiosyncratic elements to be analyzed. In addition, single sales do not permit the variance in values to be analyzed as a function of the elapsed time between sales.

Obviously, ceteris paribus, price indices estimated from equally sized samples of multiple sales are more efficient than those arising from samples of single sales.

Unfortunately, a random sample of transactions during any time interval is likely to yield a subsample of repeat transactions which is much

\(^2\) Case and Shiller (1987, 1989) specified the error structure in Eq. (6) with \( B = 0 \), finding \( A > 0 \) and statistically significant for four metropolitan areas. Abraham and Shauman (1991) and Quigley and Van Order (in press) specified the error structure in Eq. (6) and found \( A > 0 \) and \( B < 0 \) for 27 of 30 metropolitan areas and for all 5 census regions.
smaller than the subsample of single sales. For example, Abraham and Schauman (1991) found, using Freddie Mac data, that only 2.5% of transactions in 30 metropolitan areas were repeat sales over a 19-year time interval. Case and Shiller (1987) found that only 4.1% of transactions were repeat sales in four large metropolitan areas over a 16-year interval. Clapp et al. (1991) found that 25% of Connecticut transactions were repeat sales, while recent work by Case et al. (1991) found that in a 7-year period only about 14% of 12,681 transactions in Fairfax County, VA were repeat sales without changes in housing attributes.

More important than the small sample sizes for analysis of repeat sales is the possibility—that repeat sales are not a random sample of all housing sales, at least not during the short or medium run. It has repeatedly been found that dwellings which are subject to repeat sales differ in many characteristics from those which are sold only once during a given time interval. In particular, lower priced and homogeneous "starter" homes are more frequently traded than higher priced luxury accommodations. One study reports a 15% lower average price for repeat sales than for single sales of residential properties (Clapp et al., 1991). Another recent study presents an explicit model of the sample selection process which generates observations on sales in local housing markets (Gatzlaff and Haurin, 1993). The authors conclude that repeat sales indexes are likely to be quite biased.

To combine samples of single and multiple sales in a single analysis, combine Eqs. (3) and (4),

\[ V_{it} = \beta X_{it} + P_i + \xi_i + \epsilon_{it}, \]

and estimate the parameters using the subsample of repeat sales. That is, regress the log sale price on the log of housing characteristics, a set of dummy variables for time period (or perhaps some other parameterization of time), and a set of dummy variables for individual dwellings. Similarly, estimate (3) using the same sample. This yields \( \hat{\sigma}^2 \), \( \hat{\sigma}_\xi \), unbiased estimates of \( \sigma^2 \) and \( \sigma^2_\xi \) and a set of residuals \( \hat{\epsilon}_{it} \). After some slight manipulation, this also yields \( \hat{\sigma}^2_{\xi} \), an unbiased estimate of \( \sigma^2_{\xi} \).

For all multiple sales, estimate Eq. (6) with dependent variable \( [\hat{\epsilon}_{it} - \epsilon_{it}]^2 \). This yields regression coefficients \( \hat{A} \) and \( \hat{B} \), estimates of \( A \) and \( B \). Together, these parameters identify completely the variance–covariance matrix of disturbances in Eq. (7).

\[
E(\xi_i + \epsilon_{it}, \xi_j + \epsilon_{jr}) = \begin{cases} 0 & \text{for } i \neq j, \\ \hat{\sigma}^2_{\xi} + \hat{\sigma}^2_{\epsilon}[1 + \hat{A}(t - \tau) + \hat{B}(t - \tau)^2] & \text{for } i = j. \end{cases}
\]

\footnote{\( \hat{\sigma}^2_{\xi} = (\hat{\sigma}^2_{\epsilon} \cdot f - \hat{\sigma}^2_{\xi})/H, \) where \( H \) is the number of individual dwellings and \( f \) corrects for degrees of freedom, i.e., \( f = (N - H)/N, \) where \( N \) is the degrees of freedom in the computation of \( \hat{\sigma}^2_{\xi}. \)}
Now, using the entire sample of single sales and repeat sales, estimate Eq. (7) by generalized least squares, where the weights are derived from Eq. (8).

The advantage of this approach is that it utilizes all sales observations, not just repeat sales, in a common framework. The procedure utilizes the unique information on repeat sales to account for the contribution of unmeasured housing attributes to the total variance and to account for the error covariances. It utilizes the additional information on single sales to increase the efficiency of estimation of the parameters $\beta$ and $P_i$.

III. EMPIRICAL ANALYSIS

The empirical analysis is based upon a sample of 843 condominium sales recorded during the 12-year period, from January 1980 through December 1991, in downtown Los Angeles. The sample represents essentially every condominium sale within the downtown area during this time period. Condominiums were located in four different high-rise properties which realtors and real estate agents consider "comparable" for the purpose of appraisal. There are no other "comparables" within several miles of downtown. We gathered information on the original selling prices of each of the condominiums in one of the high-rise properties completed in 1980 together with all subsequent sales of these dwelling units. We also obtained information on all condominium sales in each of the other three properties beginning in 1980. Property characteristics were obtained by matching addresses to condominium floor plans. Resale information was obtained from multiple listing services, from court records, and from real estate lenders.

Because the sample consists of properties in only four high-rise buildings located within about a quarter mile of each other, the neighborhood and public service amenities associated with these properties are identical. The condominiums vary in their size and their location within each of the buildings. We recorded the date of each sale and the selling price of the property. Selling prices are reported in real terms, using the quarterly Consumer Price Index (See Economic Report of the President, 1992). In none of the condominiums were the physical characteristics of the sale properties changed during the sample period.

The 843 sales represent transactions on 584 different properties. 380 of these properties, or about 45%, were sold one time. Another 158 were sold twice, 37 were sold three times, and 9 were sold four times.

Table 1 provides a summary of the characteristics of observations on the first sales and those on subsequent sales. Note that observations on repeat sales tend to be smaller units, located on higher floors in these
condominiums. They are also substantially less valuable than those sold only once during the 12-year period. The table also indicates the characteristics of dwelling units sold one time during the period as well as those sold two, three, and four times during the 12-year interval. During a 12-year period, dwelling units involved in multiple sales constitute about 65% of all units in the sample, but their economic characteristics differ in important ways from those units involved in single sales.

The statistical analysis relates the selling prices of these apartments to the sizes, \( x_1 \), and location, \( x_2 \), of these properties and to the timing of sales. The deterministic part of the model is

\[
\log V_u = \beta_0 + \beta_1 \log x_{1u} + \beta_2 \log x_{2u} + \sum_{t=1}^{45} P_t d_{ut}. \tag{9}
\]
TABLE II
Ordinary Least Squares Regression Coefficients

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Size</th>
<th>Location</th>
<th>Time</th>
<th>Dwelling unit dummies</th>
<th>$R^2$</th>
<th>$\hat{\sigma}_1^2$</th>
<th>$\hat{\sigma}_2^2$</th>
<th>$\hat{\sigma}_3^2$</th>
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<tbody>
<tr>
<td>I</td>
<td>5.319</td>
<td>0.913</td>
<td>0.071</td>
<td>No</td>
<td>0.730</td>
<td>0.037</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(27.44)</td>
<td>(41.72)</td>
<td>(7.29)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>5.754</td>
<td>0.922</td>
<td>0.061</td>
<td>No</td>
<td>0.685</td>
<td>0.041</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(36.13)</td>
<td>(41.48)</td>
<td>(6.57)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. All sales (843 observations)

B. Repeat sales (463 observations)

| III       | 5.440 | 0.902    | 0.051| No                     | 0.742 | 0.041               | —                   | —                   |
| (21.14)   | (28.88)| (3.62)   |      |                        |       |                     |                     |                     |
| IV        | 5.818 | 0.913    | 0.052| —0.009                 | 0.680 | 0.046               | —                   | —                   |
| (25.71)   | (29.16)| (3.85)   |      |                        |       |                     |                     |                     |
| V         | 5.136 | 0.172    | 2.179| Yes                    | 0.907 | 3.231               | 0.029               | —                   |
| (0.90)    | (0.11) | (0.83)   |      |                        |       |                     |                     |                     |
| VI        | 6.027 | 0.655    | 0.761| —0.010                 | 0.879 | 0.393               | 0.031               | —                   |
| (1.13)    | (0.45) | (0.33)   |      |                        |       |                     |                     |                     |

Note: Dependent variable is the logarithm of selling price in 1980 dollars. $t$ ratios in parentheses.

* Regression includes 45 variables measuring elapsed time in quarters from January 1, 1980.

In this formulation, time, $d_{it}$, is measured in quarter years from July 1, 1980 (1980:III). The dummy variable, $d_{it}$, has a value of one if dwelling unit $i$ is sold in quarter $t$ and zero otherwise. The coefficients of the set of dummy variables, $P_t$, represent the price index for downtown condominiums.

For comparison, we also report the results using a simple exponential price trend,

$$\log V_{it} = \beta_0 + \beta_1 \log x_{i1} + \beta_2 \log x_{i2} + P_0 t,$$

(10)

where $t$ is measured in quarters.

Panel A in Table II reports the coefficients of ordinary least squares (OLS) regression estimates of Eqs. (9) and (10) based on the entire sample of 843 observations on condominium sales. Regression I explains about 73% of the variance in log selling prices during the 12-year period. The variables measuring size and location are highly significant, as is the set of 45 coefficients (not presented) which represent time. For comparison, regression II reports the results using a simple exponential price trend.

Regressions III and IV report the results of the same model estimated
on the subsample of 463 repeat sales. The precision of the parameter estimates is reduced, presumably reflecting the smaller sample size.

Regressions V and VI report the results of the OLS regressions on repeat sales incorporating the error structure of Eq. (4). Regression V includes variables representing size and location, 45 variables representing time, and a separate dummy variable for each of the dwelling units. This latter set of variables is highly significant and increases the explained variance in the model from 73 to 91%. Regression VI provides analogous estimates based on a simple exponential price trend. In both cases, when the dwelling-unit-specific dummy variables are included, the size and location variables are statistically insignificant.

Table III uses the results reported in regression V to estimate the parameters of the random walk process. In this analysis, the dependent variable is computed from the residuals in the regression. Regression VII reports the results of a linearly increasing random walk term; regression VIII estimates a quadratic. The results indicate quite strongly that the variance in housing price increases between sales, but at a decreasing rate.

Table IV presents the generalized least squares (GLS) regression results, incorporating the individual-specific error variance and the ran-

### Table III

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VII</td>
<td>( \hat{\epsilon}<em>n - \hat{\epsilon}</em>{n-1}^2 / \hat{\sigma}_1^2 = 0.0018(t - r) )</td>
<td>(7.94)</td>
<td>0.052</td>
<td></td>
</tr>
<tr>
<td>VIII</td>
<td>( \hat{\epsilon}<em>n - \hat{\epsilon}</em>{n-1}^2 / \hat{\sigma}_1^2 = 0.0050(t - r) - 0.0001(t - r)^2 )</td>
<td>(5.79)</td>
<td>0.082</td>
<td></td>
</tr>
</tbody>
</table>

*Note. t ratios in parentheses.

* Regression are based on 323 observations on differences in residuals for dwellings sold more than once.

### Table IV

Generalized Least Squares Regression Coefficients

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Size</th>
<th>Location</th>
<th>Time</th>
<th>( \hat{\sigma}_1^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IX</td>
<td>5.251</td>
<td>0.933</td>
<td>0.044</td>
<td><strong>a</strong></td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(34.40)</td>
<td>(48.14)</td>
<td>(5.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>5.802</td>
<td>0.919</td>
<td>0.045</td>
<td>-0.009</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td>(38.05)</td>
<td>(43.74)</td>
<td>(4.99)</td>
<td>(17.98)</td>
<td></td>
</tr>
</tbody>
</table>

*Note. t ratios in parentheses.

* Regression includes 45 variables measuring time in quarters from January 1, 1980.
dom walk in housing prices. We use the coefficients of the quadratic specification of the random walk, regression VIII, together with the error variances reported in Table II to compute the elements of the variance-covariance matrix, specified in Eq. (8), for the GLS regressions.

The parameters presented in Table IV differ from those in Table II, panel A, only in their efficiency of estimation. Those in Table IV utilize the additional information made available to the analyst when there are multiple observations on at least some of the dwellings in the sample.

Figure 1 illustrates the advantage of this GLS procedure when compared to the OLS estimates using the entire sample of 843 observations on condominium sales. The figure presents the 95% confidence interval for the housing price index computed from the coefficients, $P_i$, of the dummy variables in regressions I and IX. The index is normalized so that the initial price level, for 1980:III is one. The solid line represents the confidence interval associated with the OLS estimate (regression I); the broken line represents the confidence interval associated with the GLS estimate (regression IX). It is clear in this instance that the confidence interval is tighter when the information obtained from multiple sales is utilized in the estimation process. This additional information is used to estimate the individual-specific component of variation as well as the determinants of the variance in housing prices.
IV. SUMMARY

This paper has presented a simple methodology for combining information on single sales and on multiple sales in the estimation of housing price indexes. The methodology combines the advantages of large samples available in cross-sections of individual sales with the increased precision available in samples with repeated observations on individual dwellings.

The model incorporates an explicit error structure which assumes a random walk in housing prices and a dwelling-unit-specific component of variation. Estimation of the model relies upon robust linear techniques, rather than maximum likelihood estimation, to achieve asymptotic efficiency.

The technique is illustrated using a unique sample representing virtually all sales of downtown Los Angeles condominium dwellings in a 12-year period. The sample includes a large number of repeat sales, and the results suggest that recognition of this feature of the data has a substantial effect upon the precision of estimation of an index of condominium prices. In this application, the incorporation of the multiple sales nature of the data substantially reduces the standard errors and the confidence interval of the price index.

REFERENCES


