Price Formation and the Appraisal Function in Real Estate Markets

DANIEL C. QUAN
Department of Finance, University of Texas, Austin, TX 78712

JOHN M. QUIGLEY
Graduate School of Public Policy, and Department of Economics, University of California, Berkeley, CA 94720

Abstract

A real estate market model characterized by incomplete information, costly search, and varying expectations is presented. The model describes a self-selection process for market participants and a distribution of transaction prices. These transaction prices, which arise from a Nash equilibrium, can be expressed as a noisy signal, reflecting incomplete information as well as the conditions of sale. The appraiser's role is formalized as the task of signal extraction. The model emphasizes the differences in information available to individual buyers and sellers, who make transactions only infrequently, and the appraiser, whose expertise comes from observing many transactions. Based on the model, it is shown that contrary to popular perceptions, appraisal smoothing is consistent with an optimal updating strategy.

Keywords: Price dispersion, bargaining, signal extraction, appraisal smoothing

Even in light of the development of general models of market imperfections, the simple competitive market model remains the dominant tool in real estate analysis. However, despite its popularity, the competitive market model provides an inadequate portrayal of the trading environment in real estate markets. Specifically, three important features of real estate markets distinguish the price formation process from that implied by the standard model: (1) Participants in real estate markets often have incomplete information about the attributes of the purchase, and decisions to buy and sell must often be made based on this partial knowledge. (2) Given the heterogeneity and fixity of real estate, some period of costly search must be incurred by potential buyers. (3) Trades are decentralized, and market prices are the outcome of pairwise negotiations. This price determination process represents a significant departure from Walrasian auction.

This article introduces a model of real estate transactions which incorporates these three features, all of which play a crucial role in determining the eventual transaction price. The model is also useful in understanding the role of property appraisal in real estate markets and the techniques for making appraisals. This application to the appraisal function can be quite important. In real estate markets, purchases and sales of individual properties occur only infrequently. Thus the current market value of the stock of real capital, or the worth of any investor's holdings of real estate, must be inferred from limited information about recent transactions.

The theoretical underpinning of our analysis is the notion of price dispersion in market equilibrium.[1] As noted by Burdett and Judd (1983), it appears to be crucial for price dispersion in equilibrium that some *ex post* differences exist in the information available to buyers or sellers.

We take information imperfections, varying expectations, differing search costs and bargaining to be characteristic of the real estate market for both housing and investment properties. This leads to price dispersion in short run equilibrium, in which the transaction prices for identical properties vary. The model we describe includes an explicit recognition that the searching process entails not only locating the desired property but also bargaining over the price to be paid. Thus the task of buyers is one of searching for a bargaining game to provide the highest net expected payoff.

Section 1 below places the assessment problem in the broader context of real estate investment analysis. Section 2 presents the basic model of microeconomic behavior of real estate actors. Section 3 characterizes the appraisal problem more specifically. Conclusions are presented in Section 4.

1. Equity returns and estimates

Reliable market value imputations are crucial to profitability, and methods of imputation of market value are more important for real estate than for other components of investment portfolios. In the absence of sales, rates of return must be imputed rather than observed, and the correlation of returns across investment categories must be inferred from evidence on current operating income, from the sales of comparable properties, or from historical trends.

A large literature comparing real estate holdings with other investments concludes that in general, real estate has provided a somewhat higher risk-adjusted return when compared to other investment instruments; the inclusion of real estate in a portfolio of investments can substantially reduce portfolio risk; and real estate is a good hedge against inflation. Recent studies include Fama and Schwert (1977); Webb and Sirmans (1980); Miles and McCue (1982, 1984); Ibbotson and Siegel (1984); and Brueggeman, et al. (1984).

The conclusions of this entire literature depend on the construction of real estate return series which can be compared with similar indices for other investments. Many researchers have relied upon professional appraisals to represent market value (See Hartzell, et al. (1986) for a review of such studies). Indeed, this may be the standard practice. See Firstenberg, et al. (1987) for a concise and timely discussion of this issue.

The effects of appraisal technique are often "demonstrated" by identifying an exogenous process governing the behavior of "true" prices as well as describing a precise specification of appraisal methodology. For a particular methodology and for specific assumptions, it can be deduced that appraisal "smoothing" occurs.[2] This reasoning has been used to recommend procedures to correct existing appraisal data (Ross and Zisler, 1987; Geltner, 1989).

Without prior agreement on a model of real estate prices and a model of appraiser behavior, however, these conclusions arise mechanically from arbitrary assumptions made about the underlying processes.[3]

"Smoothing" results are generally attributable to reliance by appraisers on previous prices (Ibbotson and Siegel, 1984, p. 222) or previous appraisals (Ross and Zisler, 1987; Geltner, 1989) in forming their current estimates of market value. This results in some form of autoregressive structure exhibiting "inertia" and termed the "tyranny of past appraisals" (Geltner, 1989, p. 469). Note, however, that an autoregressive representation of a stochastic process does not necessarily imply smoothing.

In this model, we take a more structured view of the problem. We specify a model of price determination within a real estate market and deduce the relationship between information and transaction prices. We then ask, what is the optimal strategy for an appraiser? By assuming a quadratic loss function, we derive a parsimonious behavioral representation of the appraiser's updating procedure. The resulting model of appraisers' behavior has a natural interpretation — it can be expressed as a weighted average of a previous appraisal, as proposed by previous researchers, as well as the most recently observed transaction price. An explicit expression for the weighting parameter is thus derived.

In this model, smoothing may arise from the relative variability of general market price uncertainty and idiosyncratic transactions uncertainty, even when appraisers follow an optimal updating strategy. This is in sharp contrast to the common conjecture that smoothing arises from flaws in methodology or even incompetence among appraisers.

2. Buyers, sellers, and price determination

The model developed below describes trade among income-maximizing, heterogeneously informed agents.[4] Let P be the random price of a class of similar properties whose value is dependent on the realization of variables following a random process. There are $m + n$ agents in this market, and each of the m buyers wishes to buy one property from the n sellers. No agent observes P directly, and all transactions are based upon estimates of P, based on individual information sets. Since no agent has complete information, the buyers' and sellers' estimates, $\bar{P}^b$ and $\bar{P}^s$, deviate from P by error terms e^b and e^s, respectively:

$$\bar{P}^b = P + e^b \quad \text{and} \quad \bar{P}^s = P + e^s. \tag{1}$$

Thus, e^b and e^s summarize the extent to which each buyer and seller is informed. The errors are uncorrelated across agents and have 0 conditional means.[5]

Buyers and sellers are also distinguished by their respective discount functions, ρ^b and ρ^s, which represent the urgency of the agents to conclude a transaction. Each pair of discount parameters determines threshold or willingness to trade prices P^b and P^s:

$$P^b = (\rho^b)^{-1} \bar{P}^b \quad \text{and} \quad P^s = \rho^s \bar{P}^s. \tag{2}$$

In the absence of strategic considerations, P^b corresponds to the maximum price which the buyer is willing to pay for a given property and P^s to the minimum price at which the seller is willing to sell his property.[6] P^b is increasing in ρ^b; the more impatient the buyer is, the more he is willing to pay. The discount functions are independent of the estimation errors. Allowing for heterogeneity in both discount functions and information endowments, respective threshold price distributions $F(P^b)$ and $L(P^s)$ exist for the m buyers and n sellers.

Each buyer and seller has a reservation price P^r or offer price P^o. The reservation price is a function of the buyer's search cost and the distribution of sellers in the market. Any buyer with a reservation price derived from an optimal search rule that exceeds his threshold price will likely not participate in this market. This outcome forms a natural self-selection criterion for determining the eventual market participants. Similarly, each seller determines an offer price based on knowledge of other sellers and the distribution of reservation prices. Under certain conditions, it may not be profitable for a seller to engage in trade when the offer price is less than his threshold price. Nevertheless, it is clear that, for buyers and sellers who participate in this market, the reservation and offer prices differ from their respective threshold prices by values ϵ^b and ϵ^s:

$$P^r = P^b - \epsilon^b \quad \text{and} \quad P^o = P^s + \epsilon^s, \tag{3}$$

where ϵ^b, $\epsilon^s > 0$ represent the strategic component of the agent's overall willingness to trade prices. Thus a buyer (seller) will be unwilling to trade with a seller (buyer) with an offer (reservation) price P^o (P^r) where $P^r < P^o < P^b$ ($P^s < P^r < P^o$), since, strategically, he obtains a higher payoff, given his knowledge of the potential trades which are possible.

For a matched buyer and seller pair for whom $P^r \geq P^o$, the region between the two prices represents the surplus to be divided. Any partitioning of this surplus is desirable for both players; here the Rubinstein (1982) noncooperative bargaining approach is adopted to specify the manner with which the surplus "cake" is divided.

It can be shown (Rubinstein, 1982; Shaked and Sutton, 1984) that such a game results in the seller getting the share

$$\omega = \frac{1 - \rho^b}{(1 - \rho^b \rho^s)}$$

and the buyer getting

$$1 - \omega = \frac{(\rho^b(1 - \rho^s))}{(1 - \rho^b \rho^s)}.$$

The equilibrium partition of the surplus depends on the relative bargaining position of the buyer and seller, as represented by each agent's discount function. For an impatient buyer, ρ^b approaches 0 and the seller receives the full surplus. Conversely, if the seller is impatient, the seller receives the share $1 - \rho^b$ and the buyer gets ρ^b.[7]

The equilibrium shares obtained in Rubinstein's perfect equilibrium correspond to the bargaining parameters in the asymmetric Nash bargaining game (see Binmore and Dasgupta, 1987). Using this result, a given transaction price, P^T, negotiated between a buyer and a seller in our model can be expressed as a weighted average of the reservation and offer prices:

$$P^T = \omega P^r + (1 - \omega)P^o, \tag{4}$$

where ω is the equilibrium share. The condition of the sale can thus be represented by the relative urgency with which the buyer and seller wish to conclude a transaction. In a "seller's market," ρ^b is small. From (4), as $\rho^b \to 0$, $\omega \to 1$ and $P^T \to P^r$, the seller extracts an increasingly large surplus.

2.1. The buyer's behavior

In this section we characterize the decision of buyers to enter the market and their subsequent search behavior. The distribution of reservation prices arises from the precommitment to a negotiating strategy. Since the employment of such strategies is common knowledge, search is conducted over a set of possible noncooperative games with the perfect equilibrium partitions representing the search payoffs. In this way, a realistic element of real estate transactions is captured.

The buyer is assumed to have knowledge of the payoff distribution but not to know the location of any individual seller. The perfect equilibrium surplus partitions are functions of P^r, P^o, ρ^b and ρ^s. In our model, we consider the simpler case whereby ρ^s and ρ^b are fixed and known by all buyers and sellers.[8]

A buyer searches for the game which yields the largest surplus by sampling from the distribution of sellers and their offer prices, defined as $H(P^o)$. This search will continue until the marginal expected gain from obtaining an extra observation is equated with the marginal cost of search. Under well known conditions (see Lippman and McCall, 1976; Weitzman, 1979), an optimal stopping rule exists and the searching procedure has a reservation price property expressed in terms of surpluses. If the buyer engages in a game with a seller which provides him with a payoff below the reservation level, the buyer will continue searching. If the negotiated surplus is above, the buyer will stop searching and will conclude a transaction at the specified surplus partition.

Specifically, consider an elementary sequential search model with an infinite horizon, no discounting and a known distribution $H(P^o)$. Upon meeting, each actor learns of the counterpart's discount parameters as well as the reservation and offer prices. A trade is feasible if the buyer's reservation price is larger than the seller's offer price. If a trade is feasible, the surplus is divided according to the rules noted above. If a trade is not feasible the buyer rejects the seller's offer and selects another independent draw from the distribution of sellers.

Each buyer's surplus partition, S_i, associated with meeting and negotiating with seller i can be expressed as:

$$S_i = \begin{cases} (1 - \omega)(P^b - P_i^o) & \text{if } P^b > P_i^o \\ \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The search to maximize surplus is equivalent to search for the lowest (P^o). From (5), the objective function is

$$E[\Pi] = (1 - \omega)(P^b - E\min(P_1^o, P_2^o, P_3^o, \ldots, P_n^o)) - nc, \tag{6}$$

where n is the random stopping time, c is the cost per search, and recall is permitted. Defining P^r as the reservation price, the expected gain from search as dictated by the best stopping rule is:

If $P_i^o \leq P^r$, buy the property and receive S_i.
If $P_i^o > P^r$, do not buy and continue search.

It can be shown that the solution to (6) implies an equilibrium relationship between the search cost and the reservation price:

$$\frac{c}{1 - w} = \int_0^{P^r} (P^r - P^0)dH(P^0) = \int_0^{P^r} H(P^0)dP^0 \equiv D(P^r). \tag{7}$$

$D(P^r)$ is an increasing function of P^r, reflecting the fact that those individuals with high search cost will have high reservation prices. The reservation price relationship, which allows for bargaining, differs from the conventional search problem in the denominator of the left-hand side term. Since $0 < \omega < 1$ for a reasonable range of discount parameters, the reservation price with bargaining will be larger in the absence of such friction. This arises because sellers are not passive price takers, and consequently, buyers cannot extract all the surplus. Buyers must therefore be satisfied with sellers who have higher threshold prices — leading to smaller surpluses. When the seller is in a stronger bargaining position, the buyer's reservation surplus level becomes smaller. Conversely, in a buyer's market, $\rho^s \to 0$, $\omega \to 1 - \rho^b$, and the buyer's reservation price will be determined by (6). Unless $\rho^s = 0$ and $\rho^b = 0$, the reservation price in this framework will always be larger than in the absence of bargaining; the standard result is a limiting case.

The self-selection mechanism determining the eventual market participants from the set of m potential buyers for a given property is straightforward. We define a market *participant* as an individual with a threshold price and search cost pair (P^b, c) such that $P^r \leq P^b$.

First, consider the set of m^* market *participants*. If any member draws an offer price P^o such that $P^o \leq P^r$, a purchase will take place. If offers are drawn such that $P^r < P^o \leq P^b$, no purchase will be observed. If an individual continues searching, a lower price can be obtained given search costs and the knowledge of $H(P^o)$. Clearly, no purchase takes place for all $P^o > P^b$.

Clearly, the $m - m^*$ group of potential buyers with (P^b, c) pairs, such that $P^r > P^b$ will not participate in the market (since their expected gains from searching are less than their valuations of the property).[9]

Define $F^*(P^b)$ and $G^*(P^r)$ as the distribution of threshold and reservation prices for the m^* self-selected market participants, respectively. Clearly, for all P^b and P^r in the support of $F^*(P^b)$ and $G^*(P^r)$, $G^*(P^r) \leq F^*(P^b)$. That is, the relationship between $G^*(P^r)$ and $F^*(P^b)$ is one of first-order stochastic dominance.[10]

This definition of a participant gives a necessary condition for an eventual transaction. Market participants who satisfy this condition need not necessarily conclude transactions. However, any observable transaction price must be bounded at the top by the set of reservation prices.

The reservation price can thus be expressed as a function of the search cost. Let $Q(c)$ and $G(P^r)$ be the distribution of the search cost and the reservation price respectively. Then

$$G(P^r) \equiv Pr(R \leq P^r) = Pr[c \leq (1 - \omega)D(P^r)] = Q[(1 - \omega)D(P^r)], \qquad (8)$$

where R is a random variable.

2.2. The seller's behavior

In this section, we define the seller's informationally determined threshold prices and characterize their optimal offer prices. If sellers are Nash players with respect to other sellers, their optimal responses to a costly search by buyers will result in a dispersion of offer prices, even for identical properties. We define an equilibrium distribution of offer prices in terms of sellers' expected profits.

It is convenient to express the buyer's search strategy in terms of the cumulative distribution of the offer prices. Note from (7) that

$$\frac{dD(P^r)}{dP^r} = \int_0^{P^r} h(P^o)dP^o = H(P^r), \qquad (9)$$

where, once again, $H(.)$ is the offer price distribution. From (8), the density of the reservation price is:

$$g(P^r) = (1 - \omega)q[(1 - \omega)D(P^r)] \frac{dD(P^r)}{dP^r} \qquad (10)$$

$$= (1 - \omega)q[(1 - \omega)D(P^r)]H(P^r).$$

Consider the actions of the sellers. Each seller realizes that, by lowering price, profit conditional upon sale will be lower, but the pool of potential buyers will increase, since additional buyers with lower reservation prices will be induced to search. Conversely, by raising price, the revenue conditional upon sale will be higher, but the pool of potential customers will be smaller. Equilibrium is characterized by the expected zero profit condition; each seller who sets a price from the equilibrium distribution of offer prices will earn the same level of expected profits. Consider the behavior of the nth seller, given that the other $n - 1$ sellers have already chosen their offer prices. Using a derivation similar to Rob (1985), it is shown in Appendix 1 that the expected profit function of a seller with a given P^o will have the following expected profit function:

$$\Pi(P^o) = (1 - \omega) \int_{P^o}^{\infty} (\omega P^r + (1 - \omega)P^o)q[(1 - \omega)D(P^r)]dP^r. \qquad (11)$$

The maximization of (11) with respect to P^o summarizes the best response of this seller, given the behavior of the buyers and all other sellers. Let K be the common level of

profits for all sellers who behave this way. Then an equilibrium distribution, $H(.)$, is one in which $\Pi(P^o) \leq K$ for all P^o, while the equality holds for all P^o in the support of $H(.)$. Note that, although all sellers will earn the same level of profits in equilibrium, no seller is indifferent to the price selected from the equilibrium distribution. This is due to the fact that decisions to raise or lower prices will affect not only the pool of potential customers but also the number of sellers at a given price. This, in turn, will affect probabilities of sale and expected profits.

As is the case with buyers, each seller's decision to sell is based on a comparison of threshold and offer prices, and no potential seller will enter the market if his offer price is lower than his threshold price. Thus a market *participant* is a seller for whom $P^s \leq P^o$.

Let n^* be the number of sellers from the n potential sellers who are market participants. Define $L^*(P^s)$ and $H^*(P^o)$ as the distribution of threshold and offer prices, respectively, for these n^* sellers. By analogy to the argument in 2.1, it follows that $L^*(P^s) \leq H^*(P^o)$. That is, the relationship between $L^*(P^s)$ and $H^*(P^o)$ is one of first-order stochastic dominance.

2.3. Transaction prices

The previous sections indicate bounds for the reservation prices for buyers and the offer prices for sellers. In this section, we use this information to derive the distribution of prices from which an observable market transaction must be drawn.

For the risk-neutral income maximizers in this market, it was shown that the transaction price, P^T, can be expressed as the Nash bargaining solution for a given bargaining parameter, ω. For distributions of reservation prices for the buyers and offer prices for the sellers, the feasible set of prices from which a transaction price is drawn is the convolution of these distributions. Define the density of the convolution of the two distributions as $k(P^T)$. For the case in which P^o and P^r are independent, the density of P^T is:

$$k(P^T) = \int_0^{P^T} h^*[(1 - \omega)P^o]g^*[P_T - (1 - \omega)P^o]dP^o. \tag{12}$$

Thus, a transaction price is defined as a price drawn from the density $k(P^T)$.

2.4. Price determination: A simple example

In this section, we present a simple graphic example. Let the density function of the reservation prices for the group of self-selected buyers be characterized by a gamma function suitably normalized for the price interval $0 \leq P \leq 4$.

$$g(P^r) = \frac{\lambda^2 P^r e^{\lambda(4-P^r)}}{(e^{4\lambda} - 4\lambda - 1)}. \tag{13}$$

The offer price distribution $h(P^o)$ consistent with the above reservation price distribution can be obtained by the expected profit function as shown in (11). As discussed in the previous section, price dispersion will exist in equilibrium when sellers earn the same expected profit levels K for all prices in $h(P^o)$. Differentiating (11) with respect to P^o we obtain the following condition:

$$\int_{P^o}^{\infty} \frac{g(P^r)}{H(P^r)} \, dP^r = \frac{P^o g(P^o)}{(1 - \omega) H(P^o)}. \tag{14}$$

This condition must be satisfied by $H(.)$ to be consistent with dispersion. For tractability reasons, we consider the simpler case of $\omega = 0$; that is, sellers are price takers and buyers receive the complete surplus upon a transaction. In this case, (14) simplifies to:

$$H(P^o) = \frac{(P^o)^2 g(P^o)}{K}. \tag{15}$$

For $H(P^o)$ to be a proper distribution function, $dH(P^o)/dP^o \geq 0$ for all P^o within the bounds $P^o_1 \leq P^o \leq P^o_u$. Thus, a necessary condition for the existence of a dispersion of offer prices is

$$\frac{g'(P^o)}{g(P^o)} \geq \frac{-2}{P^o} \quad \text{for all } P^o_1 \leq P^o \leq P^o_u. \tag{16}$$

This first order difference equation implies

$$g(P^o) \geq \frac{P^o_1 P^o_u}{(P^o)^2 (P^o_u - P^o_1)}. \tag{17}$$

Thus, for a given price range, (17) provides a lower bound for the feasible reservation price density function, which supports a proper offer price distribution. As Rob (1985) has pointed out, for price dispersion to occur, it is necessary that there be a number of buyers with high reservation prices (thus making it profitable for sellers to set correspondingly high offer prices.) In a concrete example presented below, this restriction is seen to imply parameter restrictions.

From (17), it follows that only gamma distributions with $\lambda \leq .75$ will result in a nondecreasing distribution of offer prices. From (15), the seller's offer price density function is:

$$h(P^o) = \frac{(P^o)^2 (3 - \lambda P^o) e^{\lambda(4 - P^o)}}{64}. \tag{18}$$

Given these threshold price distributions for buyers and sellers, the feasible set of transaction prices, P^T, is of the form

$$k(P^T) = \frac{\lambda^5 (P^T)^4 e^{\lambda(4 - P^T)} (5 + \lambda P^T)}{(32)^2 \lambda^5}. \tag{19}$$

Figure 1 presents a schematic of the two price distributions, equations (13) and (18), drawn for $\lambda = .75$. Figure 2 shows the resulting distribution of observable transaction prices for the same parameter value.
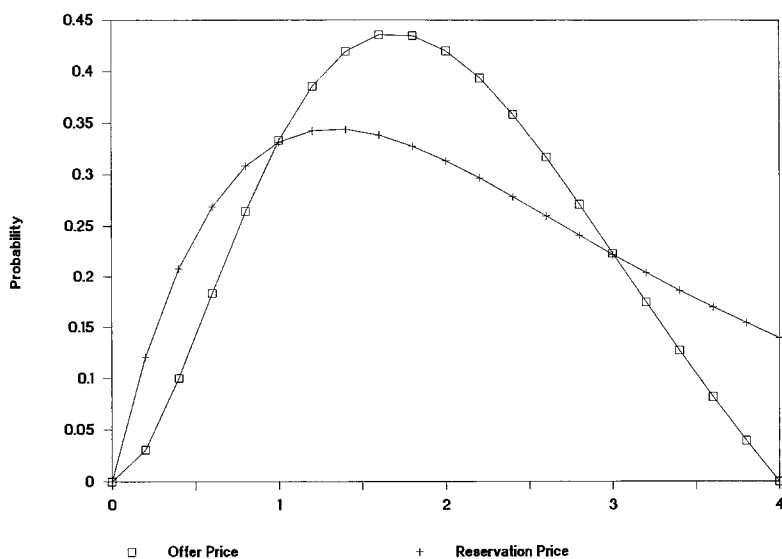

Figure 1. Reservation and offer price distribution ($\lambda = .75$).
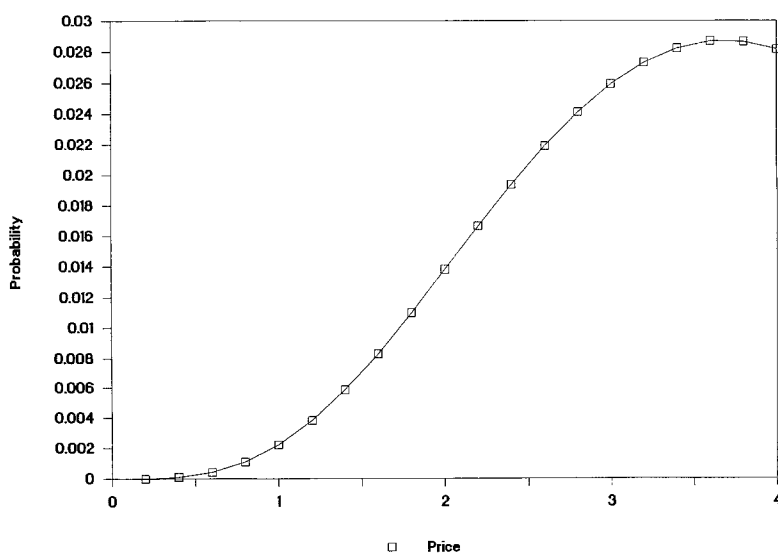

Figure 2. Transaction price distribution ($\lambda = .75$).

3. Information content of prices

This model of price formation allows us to characterize the information provided by a given transaction to an external observer. Since any transaction price is a weighted average of the reservation and offer prices, which are related to their threshold prices by (3), the relationship between P^T and P, the full information price, can be established:

$$P^T = \left[\frac{\rho^b(\rho^s - 1) + 1}{\rho^b}\right] P + \frac{(1 - \rho^b)(e^b - \rho^b\epsilon^b) + (\rho^b)^2(1 - \rho^s)(\rho^s e^s + \epsilon^s)}{\rho^b(1 - \rho^b\rho^s)} \quad (20)$$

$$= \mathbf{B}P + \nu.$$

Consider an external observer, whom we label a, who is less informed than either the buyer and seller; that is I^a, a's information set, is a proper of subset of I^b and I^s, information sets of the buyer and seller, respectively. We assume that this individual observes the condition of sale but does not observe each agent's estimation error, e^b and e^s, nor the strategic parameters, ϵ^b and ϵ^s. This agent treats these parameters as random variables. We assume that he views each transaction price as being derived from some drawing from the distribution of strategic parameters; his prior information has no power in predicting the outcome of any draw. That is, we assume independence of ϵ^b, ϵ^s, and e^a.

Under these conditions, P^T is informative about the full information price, P, for agent, a, if P^T is correlated with P, conditional on a's information set, I^a. From the definition of e^a and using (20), it follows that the conditional covariance between P^T and P is:

$$\text{Cov}(P^T, P | I^a) = E[(P^T - \mathbf{B}P^a - E(\nu|I^a))][e^a]. \quad (21)$$

Since a is less informed than both the buyer and seller, by the law of iterative expectations, $E[e^b|I^a] = E[e^s|I^a] = 0$. Independence of ϵ^b, ϵ^s and e^a simplifies (21) to:

$$\text{Cov}(P^T, P | I^a) = \mathbf{B}\text{Var}(e^a|I^a) \quad (22)$$

(See also, Appendix 2).

P^T is not informative if either $\text{Var}(e^a|I^a) = 0$ or $\mathbf{B} = 0$. The former is the case when the observer has complete information. The information content therefore rests upon whether or not $\mathbf{B} = 0$.

From (20), it is clear that for $0 < \rho^s$, $\rho^b < 1$, $\mathbf{B} > 0$. Thus, the transaction price, P^T, will always be a useful signal for the purposes of inferring P. In a real estate market characterized by bargaining and search frictions, under a reasonable range of buyer and seller discount parameters, those frictions do not dominate the information content of prices. This result permits us to characterize the optimal updating strategy of an appraiser who is nothing more than a less informed external observer of market prices.

4. The appraiser's problem

These results can be used to characterize real estate appraisal, the procedure which uses an observed transaction price, P_t^T, at time t to estimate the transaction price for "comparable" properties.

In what follows, we consider the general case in which the mean adjusted true price at time t, P_t, follows a random walk.

$$P_t = P_{t-1} + \eta_t, \tag{23}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$ and $E(\eta_t\eta_{t-j}) = 0$ for all j. Volatility in prices arises from exogenous market movements.

As noted in equation (20), the transaction price can be expressed in terms of the true price plus some "noise" terms. Consider the appraiser's problem: to estimate the market clearing price of a property by using the knowledge of the sale price, P_t^T, of an identical property. This sale price is equal to the unobservable true price plus some terms which reflect the cost of search, the condition of sale and the relevant distribution parameters. The appraiser thus must extract the relevant signal from the "noisy" transacted price. The difficulty in "filtering" such information is that the transaction price is subject to market-wide noise as well as idiosyncratic transaction noise. For this reason, the appraiser's updating rule will be a function of the relative size of these sources of disturbances.

Consider the appraiser's updating rule. Let $I_{t-1} \equiv \{P_1^T, P_2^T, P_3^T, \ldots, P_{t-1}^T\}$ be the set of all previously observed transaction prices available to the appraiser. The conclusion does not change if we let $\mathbf{B} = 1$.

$$P_t^T = P_t + \nu_t. \tag{24}$$

For convenience only, let ν_t be an i.i.d normal random variable with variance σ_ν^2 and mean 0.[11] The optimal updating procedure for an appraiser, given an initial information set I_{t-1} and an additional piece of information, P_t^T, is the so-called least squares or recursive projection

$$E[P_t|P_t^T, I_{t-1}] = E[P_t|I_{t-1}] + K[P_t^T - E(P_{t-1}|I_{t-1})]. \tag{25}$$

(See Sargent, 1979, chapter 10; Samuelson, 1965, 1973; or Chow, 1984.)

The new appraisal is made by augmenting the current appraised value of an identical ("comparable") property by some weighting, K, of last period's prediction error. The last term of (25) is the updating component. We define $P_t^* = E[P_t|P_t^T, I_{t-1}]$ as the appraiser's estimate of the market price at time t. It is shown elsewhere (Quan and Quigley, 1989) that the resulting estimate for the parameters of this model has the following form:

$$P_t^* = KP_t^T + (1 - K)P_{t-1}^* \tag{26}$$

where

$$K = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\nu^2}.$$

Appraisal proceeds through computation of a weighted average of the price recorded for the last transaction and the appraiser's previous estimate, with the weights depending

on the second moments of the error distributions. This result is intuitive, since the informational content of the system is summarized in its variance.[12] If σ_ν^2, the transaction noise, is large relative to σ_η^2, the market-wide noise, then K will take on small values and the appraiser will put more weight on the previous estimate. Thus, if the variability of prices due to condition of sale is large relative to the marketwide noise, then appraisers will rely more heavily on the previous estimate, rather than on the most recently observed transaction price. Conversely, if the variation in transaction prices due to the condition of sale is small relative to the marketwide variation, then the appraiser should place more emphasis on the transaction price.

We can now formalize the concept of appraisal smoothing. Following Geltner's derivation (1989, footnote 3, p. 470), returns are represented as first differences of the prices. Let $r_t^* = (P_t^* - P_{t-1}^*)$ be the return based on the estimated prices. Expressing the updating rule (26) in terms of returns, we obtain

$$r_t^* = K(P_t^T - P_{t-1}^T) + (1 - K)r_{t-1}^*. \tag{27}$$

Using (24) and the random walk of prices, $P_t^T - P_{t-1}^T = \eta_t + \nu_t - \nu_{t-1}$. The first term η_t is the unobservable true random return. $\nu_t - \nu_{t-1}$ is the difference in forecast error due to information differences between time t and $t - 1$. By suitable choice of the unit time interval, this error difference is small relative to the market movement noise, η_t. Ignoring this term, the expression for r_t^* is:

$$r_t^* = K\eta_t + (1 - K)r_{t-1}^*. \tag{28}$$

The appraisal-based return has the form of a first order autoregressive process with parameters K and $1 - K$ and with variance:

$$\text{Var}(r_t^*) = \frac{K\sigma_\eta^2}{2 - K} \quad \text{where} \quad K = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\nu^2}. \tag{29}$$

This expression demonstrates the relationship between the variance of the appraisal-based return and σ_η^2, the variance of the true return. From the definition of K in (29), $0 \leq K \leq 1$. If σ_ν^2 is large relative to σ_η^2, the market wide disturbance, K, is a small number and more weight is placed on the appraiser's previous estimate. Thus, the variability of returns based on appraisals will exhibit a strong reliance on previous estimates and will be "smoothed."

An alternative behavioral interpretation of (29) indicates why appraiser's estimates may result in smoothed return estimates. If σ_ν^2 is interpreted as the appraiser's subjective valuation of uncertainty as to the condition of sale, a large σ_ν^2 corresponds to the appraiser having little information about the circumstance of the transaction. If this uncertainty is large relative to the market-wide movement in prices, it is optimal for the appraiser to place less weight on the transaction in question and to rely more heavily on the previous price estimate. This reliance will introduce inertia into returns. Note that this is the behavior of appraisers who follow an optimal updating strategy. This contradicts the conventional

wisdom which asserts that smoothing by appraisers arises from flaws in methodology, poor appraisal practice, or even incompetence. When an appraiser who pursues an optimal updating strategy is faced with uncertainty about the nature of the most recently observed transaction, it is reasonable to "discount" that transaction and to rely more heavily on information acquired in previous periods. This results in smoothed return estimates made by competent appraisers.

It is clear from (29) that $\text{Var}(r_t^*)$ depends on the relative variability of the market movement noise, η_t, and the transaction noise, ν_t:

$$\text{Var}(r_t^*) = \frac{\sigma_\eta^2 \, \sigma_\eta^2}{\sigma_\eta^2 + 2\sigma_\nu^2} . \tag{30}$$

Variability in η_t arises from factors which dictate general market price movements; changes in ν_t represent variability in the agent's information set and the condition of sale. Thus, for any measured variance of appraisal-based return indices, such a measure can arise from specific combinations of market movement and transaction noise as dictated by (30). A plot of (30) is provided in Figure 3.

5. Conclusion

This article has presented a complete model of price determination in the real estate market in which property appraisal performs an important, efficiency-enhancing role. Profit-oriented, but imperfectly informed, actors in the market make varying offers to buy and sell properties, leading to a short run equilibrium in which there is some distribution of
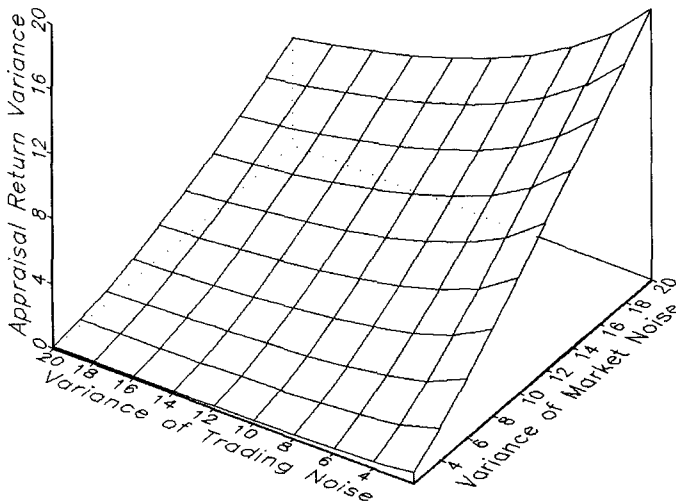

Figure 3. Variance plot.

market prices for identical properties. The role of the appraiser is to provide information so that the variance of this price distribution is reduced. The appraiser does this by updating the current estimate of the value of comparable properties every time a transaction is observed. Under quite general conditions, it can be shown that the recursive process linking appraisers to potential buyers and sellers of property reduces the market imperfections which arise, for example, from costly search and uncertain income projections by market actors.

The key to the model is the updating rule which the appraiser employs to extract the price signal from the "noisy" transactions made by imperfectly informed actors in the market. This rule specifies the appropriate weighting of the information in a given transaction with the stock of prior information available to the appraiser. This stock of information is the experience and human capital of the appraiser, which forms the basis for signal extraction.

The stylized model emphasizes the differences in information available to individual buyers and sellers, who make transactions only infrequently, and the appraiser, whose expertise comes from observing many transactions. The model indicates that, although no actor is fully informed, in a stationary world the dynamics of the market lead to a convergence of transaction prices.

The model can clearly be generalized to more realistic circumstances. For example, idiosyncratic aspects of buyers or sellers (e.g. "distress sales") can be introduced (by imposing some distribution on ϵ); excess supply or demand in local markets can be modeled by modifying the convolution equations appropriately. Finally, the optimal updating rule can be made more realistic by employing a full fledged Kalman filter algorithm. (See Kalman, 1960, or Anderson and Moore, 1979.) Indeed, it appears that these theoretical notions can be used to improve practice in the computerization of the appraisal function.

Clearly, this analysis is only a first step in relating the actions of real estate appraisers to the economics of information.

Notes

1. A variety of theoretical models have been developed which lead to such dispersion. Examples include Reinganum (1982), based upon firms' varying production costs; Salop and Stiglitz (1976), based upon differing search costs; and Wilde and Schwartz (1979), based upon consumers' inherent propensities to search.

2. For example, it is commonly asserted by researchers that appraisal data are subject to "smoothing" by the application of professional rules-of-thumb, thereby reducing the variance of the prices reported for a sample of appraisals relative to a sample of the actual sales of identical properties. If true, such an assertion is disquieting, since all measures of risk, as well as the diversification potentials of assets, are based on measures of dispersion. Thus, reliance upon an artificially smoothed series will necessarily underestimate the riskiness of an asset, and will distort the correlation of its return with the returns to other assets as well.

3. It is not difficult to find a smoothed representation of any arbitrary stochastic process, but one can with equal ease specify an appraisal strategy which can result in a larger variance.

4. Other factors which provide important motives for trade in real estate markets (such as, income shocks, differences in tastes, or planning horizons) are not developed in the interest of parsimony. These factors can be introduced to the model without significantly altering the basic results. Note that any one of these sources of heterogeneity is sufficient to insure that the so-called "no trade" results do not apply (See Milgrom and Stokey, 1982).

5. Given the common value feature of real estate in this model, we will only address the issue of learning on the part of an appraiser in this model. In principle, since each agent's estimates are correlated, interaction

between agents will alter their information sets as they meet their counterparts. This aspect of learning by the agents is ruled out in our model but is developed elsewhere in Quan (1990). The main results of this model are, however, preserved in this simpler setting.

6. A more general interpretation of this formulation can be made if we interpret $\bar{P}^b = E[P|I^b]$ (where I^b is the buyer's information set and E is the expectation operator) as the utility level the buyer receives for the property. Fishburn and Rubinstein (1982) showed that, under general conditions (where preferences defined over their valuation are continuous, reflexive, transitive and stationary and where time is valuable), the buyer's time preference can be represented by a utility function of the form $(\rho^b)^{-1}\bar{P}^b$. Under those conditions P^b can be viewed as the utility level, measured in monetary terms, that the buyer assigns to the property.

7. The asymmetry of this result reflects the "first move" advantage which is characteristic of such games. It can be shown that, as the time period between offers becomes small, this advantage becomes negligible (Binmore and Dasgupta, 1987). Even though alternate structures for such a game can circumvent such an advantage, we will retain this asymmetry for the sake of realism, since the seller typically provides the initial asking price in real estate markets.

8. Since ρ^s and ρ^b determine ω, the most natural interpretation of this assumption is that the "condition of sale" for properties is known. A more realistic case, in which the distribution of ρ^s and ρ^b are known, is considered in Appendix 3.

9. A simple example demonstrates this relationship. If $H(P^o)$ has a uniform distribution between the bounds 0 and b, then (7) can be expressed as:

$$\frac{c}{1-\omega} = \int_0^{P^r} \frac{P^o}{b} \, dP^o \text{ for } P^r < b.$$

Any potential buyer whose cost of search exceeds a critical cost c^* will not be a participant in this market where

$$c^* = \frac{(1-\omega)(P^b)^2}{2b}.$$

10. If this were not true, then a participant would exist whose reservation price was larger than his threshold price. But such an individual could not be a participant and therefore could not be a member of m^*.

11. Assuming the estimation errors have a mean of 0 simplifies the presentation. The more general case of nonzero unconditional mean is considered in Quan and Quigley (1989) as well as the case in which errors made by subsequent agents are reduced over time. Also, the normality assumption is not binding if we restrict ourselves to linear filtering rules (See Anderson and Moore, 1979, chapter 3).

12. This expression for an appraiser's updating procedure is very similar to the one hypothesized by Geltner. If the most recently observed transaction price is the appraiser's present estimate, then the signal-to-noise ratio specified above is identical to Geltner's unspecified "confidence" parameter.

Appendix 1

Following Rob, the reservation price distribution has the following discrete approximation:

$$\Delta G(P_i^r) = g(P_i^r)\Delta P_i^r = 1/m. \tag{A1.1}$$

A buyer with a P^r can only trade with a seller whose $P^o \leq P^r$. Since $H(P^o)$ is the distribution of seller offer prices, the buyer's probability of making a favorable trade is $1/nH(P^r)$. The probability of a seller with a given P^o of making a favorable match is the sum of $1/nH(P^r)$, for all $P^r \geq P^o$. If a trade is possible, the seller receives $P^T = \omega P^r + (1-\omega)P^o$. Thus, the total expected seller payoff from participating in this market is

simply the sum of the products of the probability of making favorable matches and the payoff conditional on a match:

$$\Pi(P^o) = \sum \frac{\omega P_i^r + (1 - \omega)P^o}{nH(P_i^r)} = \sum \frac{(\omega P_i^r + (1 - \omega)P^o)mg(P_i^r)\Delta P_i^r}{nH(P_i^r)}, \quad (A1.2)$$

where the last term is derived by substituting in (A1.1), and the summation is over all i for $P_i^r \geq P^o$. However, from (10),

$$g(P_i^r) = (1 - \omega)q[(1 - \omega)D(P_i^r)]H(P_i^r). \quad (A1.3)$$

Substituting in this expression into (A1.2) and taking the limit yields the following continuous case:

$$\Pi(P^o) = (1 - \omega) \int_{P^o}^{\infty} (\omega P^r + (1 - \omega)P^o)q[(1 - \omega)D(P^r)]dP^r. \quad (A1.4)$$

This expression appears as equation (11) in the text.

Appendix 2

We calculate the term $\text{Cov}(P^T, P|I^a)$:

$$\text{Cov}(P^T, P|I^a) = E[(P^T - E(P^T|I^a))][P - E(P|I^a)]. \quad (A2.1)$$

By the definition of P^T and the independence assumption,

$$E(P^T|I^a) = BE(P|I^a) - E(\epsilon^b|I^a) + E(\epsilon^s|I^a). \quad (A2.2)$$

Substituting this expression into (A2.1) we get:

$$\text{Cov}(P^T, P|I^a) = E[P^T - BE(P|I^a) - E(\epsilon^b|I^a) + E(\epsilon^s|I^a)][e^a]. \quad (A2.3)$$

Expanding the P^T term and taking expectations yields

$$\text{Cov}(P^T, P|I^a) = BE[e^a|I^a]^2 = B\text{Var}(e^a|I^a). \quad (A2.4)$$

This expression appears as equation (22) in the text.

Appendix 3

This appendix generalizes the results reported in the text to the case in which ρ^b and ρ^s vary among the population.

Consider the buyers. Suppose each buyer knows his own ρ^b and P^r but does not know ρ^s or P^o. Let i index buyers and j index sellers. If buyer i meets seller j, the surplus to buyer i is:

$$S_{ij}^b = \frac{\rho_i^b(1 - \rho_j^s)}{1 - \rho_i^b \rho_j^s} (P_i^r - P_j^o), \tag{A3.1}$$

and the seller gets:

$$S_{ij}^s = \frac{(1 - \rho_i^b)}{1 - \rho_i^b \rho_j^s} (P_i^r - P_j^o), \tag{A3.2}$$

Thus, for a buyer with given ρ^b and P^r, the distribution of ρ^s and P^o will result in a corresponding distribution of surpluses. Buyer i searches for the maximum surplus subject to his search cost by solving:

$$E\max[S_{i1}^b, S_{i2}^b, \ldots, S_{in}^b] - nc. \tag{A3.3}$$

The resulting reservation level of surplus for buyer i is labelled $\bar{S}_i^b$. It can be shown from (A3.1) that for given $\bar{S}_i^b$ ρ^b and P^r,

$$\frac{dP^o}{d\rho^s} = \frac{\bar{S}_i^b(\rho^b - 1)}{\rho^b(1 - \rho^s)^2} < 0 \quad \text{and} \quad \frac{d^2P^o}{d\rho^{s2}} = \frac{2\bar{S}_i^b(\rho^b - 1)}{\rho^b(1 - \rho^s)^3} < 0. \tag{A3.4}$$

$\bar{S}_i^b$ therefore defines an indifference level of buyer i's surplus for various combinations of ρ^s and P^o. The indifference level is downward sloping and concave to the origin. Thus, for buyer i with a given $\bar{S}_i^b$, trading with an impatient seller (small ρ^s) who has a high P^o will yield the same level of surplus as a more patient seller who has a lower P^o. The extent of this tradeoff is dependent on ρ^b.

Now we can characterize the set of sellers with whom i may trade. Let all sellers be identified by the pair (ρ_j^s, P_j^o), and let all buyers be identified by (ρ_i^b, P_i^r). Any given buyer will trade only with sellers who can provide a surplus at least as large as the reservation level, $\bar{S}_i^b$. Thus the feasible set of trading partners for i is:

$$\wp_i^b = \left\{ (\rho^s, P^o) : \frac{\rho_i^b(1 - \rho^s)}{1 - \rho_i^b \rho^s} (P_i^r - P^o) \geq \bar{S}_i^b \right\}. \tag{A3.5}$$

Given the cost of search and the distribution of ρ^b and P^r over the set of buyers, there exists a corresponding distribution of feasible trading partners for each of these buyers. Define such a distribution as $\mathcal{F}(\wp_i^b(\rho^s, P^o))$. Thus $\mathcal{F}(\wp_i^b(x, y))$ corresponds to the proportion of buyers who can trade with seller (x, y).

Now consider the sellers. Each seller j, endowed with ρ_j^s and P_j^o, knows $\mathcal{F}(.)$; trade is only profitable with buyers whose indifference level is greater than (ρ_j^s, P_j^o). The proportion of such buyers is $1 - \mathcal{F}(\rho_j^s, P_j^o)$. Thus, given n buyers, the probability of meeting a feasible trading partner by random matching is:

$$\frac{1}{n(1 - \mathcal{F}(\rho_j^s, P_j^o))} .$$ (A3.6)

The payoff upon a successful match can now be indicated. For seller j, order the buyers and index them so that buyer 1 represents the match which will result in the smallest buyer surplus; buyer 2 represents the match, yielding the next smallest surplus, and so on, for all feasible trading partners of j. That is, if we let n_j be the index of the last buyer with whom seller j can trade, then the set of feasible buyers for j is $(1, \ldots, n_j)$. Thus from (A3.1) and (A3.2), we know that if seller j meets with buyer 1, the seller will receive

$$S_{1j}^s = \frac{(1 - \rho_1^b)}{(1 - \rho_1^b \rho_j^s)} (P_i^r - P_j^o),$$ (A3.7)

and so on for all buyers until buyer n_j. The profit function for the seller will therefore be the sum of the seller surplus for each buyer whose index ranges from 1 to n_j. That is, profits Π is:

$$\Pi(\rho_j^s, P_j^o) = \sum_{i=1}^{n_j} \frac{\dfrac{(1 - \rho_i^b)}{(1 - \rho_i^b \rho_j^s)} (P_i^r - P_j^o)}{n(1 - \mathcal{F}(\rho_j^s, P_j^o))} .$$ (A3.8)

The maximization of (A3.8) with respect to the pair (ρ_j^s, P_j^o) will determine their optimal combination of (ρ_j^s, P_j^o) and thus the level of surplus which they demand. If the maximization of (A3.8) produces a pair (ρ_j^s, P_j^o) which yields a level of surplus less than that determined by each seller's endowed pair, this seller will exit the market. If it is larger, then he stays in and engages in trade.

References

Anderson, B.D.O. and Moore, J.B. *Optimal Filtering*, New Jersey: Prentice-Hall Inc., 1979.

Binmore, K. and Dasgupta, P., eds., *The Economics of Bargaining*. London and New York: Basil Blackwell, 1987.

Brueggeman, W.B., et al. "Real Estate Investment Funds: Performance and Portfolio Considerations." *AREUEA Journal* 12 (Fall 1984), 333–354.

Burdett, K., and Judd, K. "Equilibrium Price Dispersion." *Econometrica* 51 (July 1983), 955–969.

Chow, G. "Random and Changing Coefficient Models." In *Handbook of Econometrics*, ed. by Z. Griliches and M.D. Intriligator, Vol. II, Elsevier Science Publishers, 1984.

Fama, E. and Schwert, G. "Asset Returns and Inflation." *Journal of Financial Economics* 5 (November 1977), 115–146.

Firstenberg, P.M., et al. "Managing Real Estate Portfolios." *Goldman Sachs Real Estate Research Publication*, November 1987.

Fishburn, P. and Rubinstein, A. "Time Preference," *International Economic Review*, 23 (October 1982), 677–694.

Geltner, D. "Estimating Real Estate's Systematic Risk from Aggregate Level Appraisal-Based Returns." *AREUEA Journal* 17 (Winter 1989), 463–481.

Hartzell, David, et al. "Diversification Categories in Investment Real Estate." *AREUEA Journal* 14 (1986), 230–254.

Ibbotson, R. and Siegal, L.B. "Real Estate Returns: A Comparison with Other Investments." *AREUEA Journal* 12 (Fall 1984), 219–242.

Kalman, R.E. "A New Approach to Linear Filtering and Prediction Problems." *J. Basic Eng., Trans. ASME* 82, series D (March 1960), 35–45.

Lippman, S.A. and McCall, J.J. "The Economics of Job Search: A Survey." *Economic Inquiry* XIV (September 1976), 347–368.

Miles, M. and McCue, T. "Commercial Real Estate Returns." *AREUEA Journal* 12 (Fall, 1984), 355–377.

Miles, M. and McCue, T. "Historic Returns and Institutional Real Estate Portfolios." *AREUEA Journal* (Summer, 1982), 184–199.

Milgrom, P. and Stokey, N. "Information, Trade and Common Knowledge." *Journal of Economic Theory* 26 (1982), 17–27.

Quan, D., "Information, Prices and Common Value Search in Real Estate Markets." Working paper No. 90/91-2-6, Finance Department, University of Texas at Austin, 1990.

Quan, D. and Quigley, J. "Inferring an Investment Return Series for Real Estate from Observations on Sales." *AREUEA Journal* 17 (Summer, 1989), 218–230.

Reinganum, Jennifer F. "Strategic Search Theory." *International Economic Review* 23 (February 1982), 1–17.

Rob, R. "Equilibrium Price Distributions." *Review of Economic Studies* LII (1985), 487–504.

Ross, S. and Zisler, R. "Managing Real Estate Portfolios, Part 3: A Closer Look at Equity Real Estate Risk." *Goldman Sachs Real Estate Research*, November 1987.

Rubenstein, A. "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50 (1982), 97–109.

Salop, S. and Stiglitz, J. "Bargains and Ripoffs: A Model of Monopolistically Competitive Prices." *Review of Economic Studies* 44 (1976), 493–510.

Samuelson, P.A. "Proof That Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review* 6 (1965), 41–49.

Samuelson, P.A. "Proof That Properly Discounted Present Values of Assets Vibrate Randomly." *The Bell Journal of Economics* 4 (Autumn 1973), 369–374.

Sargent, T.J. *Macroeconomic Theory*, New York: Academic Press, 1979.

Shaked, A. and Sutton, J. "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model." *Econometrica* 52 (1984), 1351–1364.

Webb, J. and Sirmans, C.F. "Yields and Risk Measures for Real Estate, 1966-1977." *Journal of Portfolio Management* (Fall 1980).

Weitzman, M. "Optimal Search for the Best Alternatives." *Econometrica* 47 (1979), 641–654.

Wilde, L.L. and Schwartz, A. "Equilibrium Comparison Shopping." *Review of Economic Studies* 46 (1979), 543–554.