

# Defaults on mortgage obligations and capital requirements for U.S. savings institutions

## A policy perspective

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This paper analyzes credit risk for residential mortgages. Estimates of the hazard of mortgage default are presented, based upon a large sample of conventional loans issued in the late 1970s. Mean returns are estimated, together with variances and covariances for various loan-to-value (LTV) ratios and geographic groups. These results are then used to analyze capital requirements for institutions holding mortgages. We ask: For a given bankruptcy risk, how should capital requirements vary by LTV and geographical diversification? The empirical results indicate that both factors have powerful effects upon bankruptcy risk; thus capital requirements should vary substantially along these dimensions.

### 1. Introduction

The enormous public expenditures required in the next few years by the widespread failures of major home mortgage lending institutions in the United States (building societies or savings and loan institutions, henceforth S&Ls) will probably exceed all other government expenditures on housing in the United States. By way of comparison, it is currently estimated that the

\*A previous version of this paper was presented at the Conference on International Studies of Housing: The Effects of Government Policies, Uppsala, Sweden, June 1989. We acknowledge the research assistance of Carl Mason and Fred Schmidt and help from Jesse Abraham, Bill Schauman, and Peter Zorn. We are particularly indebted to Chester Foster for assistance in programming the capital requirements model. The paper benefited from the comments of conference participants, especially those of David Brownstone and from a careful review by Fred Balderston. Quigley's research on this project is supported by the Center for Real Estate and Urban Economics, University of California, Berkeley.

cost to U.S. taxpayers of the government guaranteed deposits in the bankrupt S&Ls will be well over \$100 billion in present value; the entire annual budget of the U.S. Department of Housing and Urban Development is currently \$15–20 billion. All Federal government expenditures on housing for low-income households (including subsidies from other welfare programs) run about \$40 billion annually.

In the light of the disastrous outcome, the various causes have become a topic for intensive ex post analysis [e.g. Barth et al. (1989)]; at the same time, a variety of proposals have been made to prevent another round of S&L failures after the next interest rate cycle. A key ingredient of these new proposals is enforcement of more stringent capital standards.

Capital proposals so far have involved adjusting capital requirements for the different perceived risks of different assets (e.g. investment in commercial real estate is thought to be riskier than is investment in single-family, owner-occupied housing). While these proposals are clearly a step in the right direction, they are not strongly influenced by either empirical or theoretical work. In particular, they generally ignore the notion of risk as a portfolio-wide (rather than asset-specific)<sup>1</sup> concept and they are not based on empirical work. This causes them to ignore some potentially important sources of risk control, such as geographic diversification.<sup>2</sup>

This paper analyzes capital requirements for institutions holding mortgages. It focuses entirely on credit risk for single-family mortgages, rather than interest rate risk or other types of credit risk. Our data consist of records on 300,000 conventional (i.e. not government-insured) loans originated from 1976 through 1980 and bought by the Federal Home Loan Mortgage Corporation (Freddie Mac). We estimate mean returns and their variances and covariances for various loan-to-value (LTV) and geographic groups. We ask the question: For a given risk of bankruptcy by the S&L how should capital requirements vary by LTV and by amount of geographic diversification?

We find that both LTV and diversification have a significant effect on the risk of bankruptcy, which suggests that policy would be improved by setting capital requirements by LTV and by geographic diversification. Our calculations imply that an S&L with loans having LTVs between 81 and 90 percent needs about one-third of the capital of an institution with LTVs between 91 and 95 percent and, holding LTV constant, a nationally-diversified institution needs only one-half to one-third of the capital of one located entirely in one region (out of five).

<sup>1</sup>This applies only to credit risk. The 1988 Federal Home Loan Bank proposal includes a separate analysis of interest risk, which does take a portfolio approach.

<sup>2</sup>See Corgel and Gay (1987) and Ogden et al. (1989) for recent work on the gains to diversification.

## 2. Models

### 2.1. Capital

The major reason for imposing capital requirements upon lending institutions is to protect the deposit insurer, who must make up losses to depositors in the event of bankruptcy. In the case of federally chartered mortgage institutions in the United States, the ultimate insurer was, until recently, the Federal Savings and Loan Insurance Corporation (FSLIC), a subsidiary of the FHLBB. In September 1989 the insurer responsibility was shifted to the Federal Deposit Insurance Corporation (FDIC).

Invested capital put up by the firm acts as a cushion for the insurer and is analogous to the 'deductible' on other types of insurance. Moreover, capital 'at risk' provides a powerful incentive for owners and managers of S&Ls to control those risks. Problems with deposit insurance and analyses of incentives and pricing have a long history.<sup>3</sup> We simply note at this point a central conclusion of this literature: a logical way to think of a capital requirement is as that amount that reduces risk by just enough to make insurance premiums 'cover' default costs, in an expected present value sense, or at least enough to limit the expected present value of losses.

Deducing a specific capital requirement from this principle is clearly a formidable problem. It involves not only modeling the firm, but also solving a complicated pricing model. The latter requires modeling the timing of bankruptcy and the magnitude of losses conditional upon bankruptcy.

We propose a simple approach to assessing capital adequacy. We postulate that capital is adequate if it is sufficient to insure that a firm will survive for a period of  $T$  years with probability  $\rho$ , and we calibrate the model so that  $\rho$  and  $T$  are consistent with the revealed preferences inherent in reform proposals.

It was proposed by the Bank Board in 1988 that home mortgages with 20 percent downpayments (80 percent LTV) or with insurance coverage have 3 percent of owners' capital at risk. We choose  $\rho$  so that 3 percent is the appropriate capital requirement for an undiversified portfolio of the riskiest of these mortgages for a 10-year time horizon. Details of this calibration are discussed below. Relative to this benchmark, we calculate capital requirements by varying diversification and LTV.

### 2.2. Default and prepayment

All modern research considers default on financial obligations from an 'options perspective'. In the mortgage market, default by a consumer amounts to exercising his option to sell the house back to the lender in

<sup>3</sup>See Kane (1985) and Merton (1977), among others, on incentives and pricing.

exchange for the mortgage instrument [see Campbell and Dietrich (1983), Vandell and Thibodeau (1985), and Foster and Van Order (1984)]. Transactions costs and the value of the borrower's reputation make the computation of the value of this option complex. Nevertheless, the key variable is the owner's equity, which is a crude measure of the extent to which the put option is 'in the money'. The key state variable that determines equity is the current value of the house.

By analogous reasoning, mortgage prepayment is an option that can be called at the discretion of the borrower. By prepaying a mortgage, the consumer exercises his option to exchange a fixed and known sum of money for the mortgage instrument [see Dunn and McConnell (1981), Follain et al. (1988), and Quigley (1987)]. Again transactions costs, expectations and exogenous reasons for residential mobility make the computation of the value of the option complex. Despite this, the key variable is the present value of the mortgage payment stream at current interest rates relative to its present value at the contract rate. Again, this is a crude measure of the extent to which the call is in the money. The key state variables are current interest rates on securities of different maturities.

Capital adequacy as described above (by  $\rho$  and  $T$ ) depends upon mean 'returns', their variances, and their covariances by asset category. 'Returns' in this context means the net income from a portfolio of mortgages (where net is the difference between the fees charged for bearing credit risk and the actual losses), and the asset categories include home mortgages written with different LTVs in different geographical regions. Capital adequacy also depends upon the extent of mortgage prepayment and the fee income arising therefrom.

The moments of the distributions of returns could be estimated in a variety of ways. The most elegant would be an estimation of the underlying structural model of the decision to default or to prepay by individual homeowners. The parameters of such a model together with the distributions of the underlying state variables could be used to reconstruct the probability distributions of default losses and prepayments. Alternatively, the rich body of data on loans originated between 1976 and 1980 and followed until March 1989 can be used to estimate the probability distributions directly, provided we accept the proposition that these are representative years.

We adopt the latter approach in this paper. This strategy has distinct advantages as well as disadvantages. On the one hand, the empirical estimates of default and prepayment lack an explicit theoretical foundation. On the other hand, however, this period experienced a wide range of economic changes (high inflation, low default periods, and low inflation, high default periods), and is probably representative enough not to require more precise theoretical restrictions. More relevant to the policy question is the fact that capital requirements set by legislation cannot be expected to vary

when certain parameters of theoretical interest change (e.g. the expected inflation rate). As a regulatory matter, the best one can expect is that the requirements conform as closely as possible with experience. Hence, we compute means, variances and covariances of probability distributions of returns without relating them to an explicit microeconomic model of optimizing behavior. In this respect, our approach is similar to applications of the capital asset pricing model, which uses historical returns and their moments without a detailed microeconomic specification.

### **3. Empirical analysis**

#### *3.1. Defaults and losses*

The Freddie Mac data on its mortgage loan purchases were matched to data on subsequent defaults and the losses resulting from foreclosure. These administrative records were not originally intended for research purposes, and a variety of assumptions were employed in merging and matching records and in verifying the underlying data.<sup>4</sup>

The available information consists of mortgage default and prepayment transactions on 30-year, fully-amortizing, fixed-rate loans of various ages and LTV categories, conditional upon survival until that age. These observations are available for each of five Freddie Mac regions for mortgages originated during the 1976–1980 period. The LTV categories are: less than 81 percent, 80–90 percent, and greater than 90 percent.

Since mortgages originated before 1976 are not included in the data base, the maximum observed age is only about 13 years (through March 1989), and mortgages in the sample were written for a duration of 30 years. This limitation in data availability is more apparent than real, however, since other empirical research indicates that prepayment and default options are typically exercised early in the life of mortgage contracts, if at all.

For each category of LTV, region, and age, we estimated the loss incurred by each mortgage default. The loss was computed as the sum of the outstanding loan balance plus all out-of-pocket costs to the lender (attorneys' fees, court costs, etc.) plus the opportunity cost of the lender's equity, from the date the lender took over a property from the borrower to its final disposition, minus the proceeds from the disposition of the asset, all expressed as a present value (discounted at 15 percent) on the date of default. The ratio of this loss to the original loan balance is the conditional loss associated with default.

Table 1 summarizes the default and loss data that have been computed. Panel A presents unweighted averages of default and loss data reported by region, year of origin, and age of loan; panel B presents summary infor-

<sup>4</sup>See Zorn (1989) for a discussion of the underlying data.

Table 1  
Summary loss data by region.

	Northeast	Northcentral	Southeast	Southwest	West	U.S.
<i>A. Unweighted annual averages</i>						
Average annual default rate ( $\times 100$ )	0.111	0.447	0.087	0.374	0.228	0.371
Average conditional loss (fraction of loan)	0.294	0.438	0.238	0.396	0.261	0.349
Standard deviation of conditional loss (fraction of loan)	0.124	0.287	0.162	0.129	0.066	0.086
Average loss (in percent)	0.034	0.207	0.022	0.163	0.055	0.137
Standard deviation of loss (in percent)	0.027	0.186	0.019	0.229	0.061	0.235
<i>B. Weighted by volume of business</i>						
Average annual default rate ( $\times 100$ )	0.132	0.502	0.107	0.357	0.255	0.432
Average conditional loss (fraction of loan)	0.316	0.522	0.236	0.396	0.260	0.351
Standard deviation of conditional loss (fraction of loan)	0.124	0.287	0.162	0.129	0.066	0.086
Average loss (in percent)	0.039	0.242	0.024	0.155	0.062	0.161
Standard deviation of loss (in percent)	0.027	0.186	0.019	0.229	0.061	0.235

mation weighted by the number of loans 'at risk' in each category (i.e. the number surviving up to the beginning of the year of duration).

As is indicated in the table, annual default rates were not trivially small in this period, averaging about 0.4 percent for the United States as a whole. The variation in default rates by region is quite substantial. Default rates in the Northcentral states were about five times as large as default rates in the Southeastern states and about four times as large as default rates in the Northeastern states. These differences reflect the credit rate risk associated with the real estate markets in each of the regions, the fortunes of the regional economies, and the loan-to-value ratios and ages of the mortgages.

Regional variations also reflect institutional differences and legal restrictions in various states. In some states, especially the Northcentral states, but also New York, legislatures and courts impose substantial impediments to foreclosure on home loan defaults. The longer time interval between default, foreclosure, and ultimate disposition greatly increases the loss rates on defaulted properties.

The loss rates on defaulted mortgages are surprisingly large. For the United States as a whole, about 35 percent of the initial mortgage loan was

Table 2  
 Hazard models of default probabilities by region,<sup>a</sup>  $H_{ijt} = \lambda_{ij} \exp(\sum \beta_t)$   
 (asymptotic  $t$  ratios in parentheses).

	Northeast	Northcentral	Southwest	Southeast	West	U.S.
LTV $\leq$ 80%						
observations	15,462	16,280	20,962	5,870	111,208	169,782
defaults	22	178	148	12	484	844
80% < LTV $\leq$ 90%						
observations	13,612	16,464	17,610	4,560	58,392	110,638
defaults	96	536	408	22	1,070	2,132
LTV > 90%						
observations	13,738	7,624	18,124	6,000	11,428	56,914
defaults	134	564	936	68	520	2,222
Origin year coefficients, $\beta_t$ :						
1976	-1.751 (4.90)	-3.376 (4.72)	-3.605 (12.26)	-9.235 (0.22)	-6.574 (6.54)	-3.941 (19.40)
1977	-1.300 (4.23)	-2.287 (11.49)	-2.613 (15.37)	-0.239 (0.38)	-4.398 (17.31)	-2.839 (31.90)
1978	-0.955 (3.51)	-1.205 (9.56)	-1.552 (10.80)	-2.356 (5.36)	-2.650 (25.73)	-1.721 (30.07)
1979	-0.667 (2.70)	-0.548 (4.64)	-0.783 (5.26)	-1.231 (3.98)	-1.379 (19.99)	-0.992 (19.44)
$\chi^2$	33.42	241.85	505.28	60.86	2040.42	2357.23

<sup>a</sup> $H_{ijt}$  = hazard for LTV class  $i$ , age  $j$ , and origin year  $t$ .  $\lambda_{ij}$  = baseline hazard for LTV class  $i$  and age  $j$ .

lost upon default of a mortgage during this period. As suggested above, a large part of this loss is accounted for by the considerable lags involved. On average, it takes almost 2 years from the time a homeowner stops payment until the home is sold. (This can cost 20–25 percent of value in interest alone.) Hence, even with insurance, typically covering only the first 25 percent of loss, the net cost to the lender can be significant. As is discussed below, high LTV loans with insurance have higher default costs than low LTV loans without insurance. While the loss per default is smaller (as a result of insurance) for the higher LTV loans, these loans default much more frequently.

Together, variations in default rates and conditional losses yield the average losses reported in the table. For the nation as a whole, the average annual loss associated with a mortgage originated between 1976 and 1980 was 0.13–0.16 percent of the contract amount. The average loss varied by a factor of 10 among the five regions, from 0.02 percent in the Southeast to almost 0.25 percent, on average, in the Northcentral 'rust belt'.

Table 2 presents estimates of proportional hazard models for each of the five geographical regions and for the United States as a whole. The

dependent variable is the hazard of default at each age (the probability of default, conditional on survival to that age). The baseline hazard varies by age, separately for each LTV class.<sup>5</sup>

Specifically, we assume that the hazard,  $H_{ijt}$ , the probability of default for a loan in LTV class  $i$ , which has survived up to age  $j$  and was originated in year  $t$ , is given by

$$H_{ijt} = \lambda_{ij} \exp(\sum \beta_i t). \quad (1)$$

The baseline term,  $\lambda_{ij}$ , indicates the age profile of defaults for the excluded vintage, given LTV. The profile is shifted for different origination years by the exponential terms.

Separate parameters are estimated for each of the four origin years, 1976–1979, relative to 1980, for each region. Each of the hazard models is highly significant, and the coefficients exhibit a similar pattern in five of the six samples. Compared with mortgages issued in 1980, those issued earlier have a significantly lower hazard of default. In all regions outside of the Southeast, there is a monotonic increase in the hazard of default for mortgages issued later. An intuitive explanation for this pattern of defaults on fixed rate mortgages is found in the course of the macro economy during this period. Holders of fixed-rate mortgages issued earlier benefited more from the high levels of inflation in the late 1970s, which reduced the real value of payment streams and also generated substantial homeowner equity. This inflation benefit proved even more valuable in reducing the higher default risks expected in the early 1980s as a result of the major recession.

Fig. 1 illustrates the survival probabilities predicted by the hazard models. For the Western region, it presents the estimated survivor rates for mortgages in each of the three LTV classes originated in 1980. For these computations, the baseline hazard (which varies separately by LTV class) is estimated by the Meier–Kaplan method [see Kalbfleisch and Prentice (1980)]. Quite clearly, at higher LTVs loans are substantially more risky.<sup>6</sup>

Both default rates and loan losses clearly vary by region and LTV, and the pattern of these outcomes varies by age. Presumably some risks can be modified with regional diversification.

Tables 3 and 4 summarize the degree of independence in the default and

<sup>5</sup>In the jargon of hazard or reliability theory, this model is 'blocked' by LTV class.

<sup>6</sup>We also estimated logit models of the probability of default separately by region and LTV class. These models document the decline in the conditional default probabilities for more seasoned mortgages. The conditional default logit declines with the age of the mortgage, and the decline is slightly less than proportional. In common with the results from the more appropriate hazard models, the logit models highlight the differences in defaults for mortgages originated in different years throughout this period. Mortgages issued later have significantly higher default rates. Copies of these results are available from the authors.

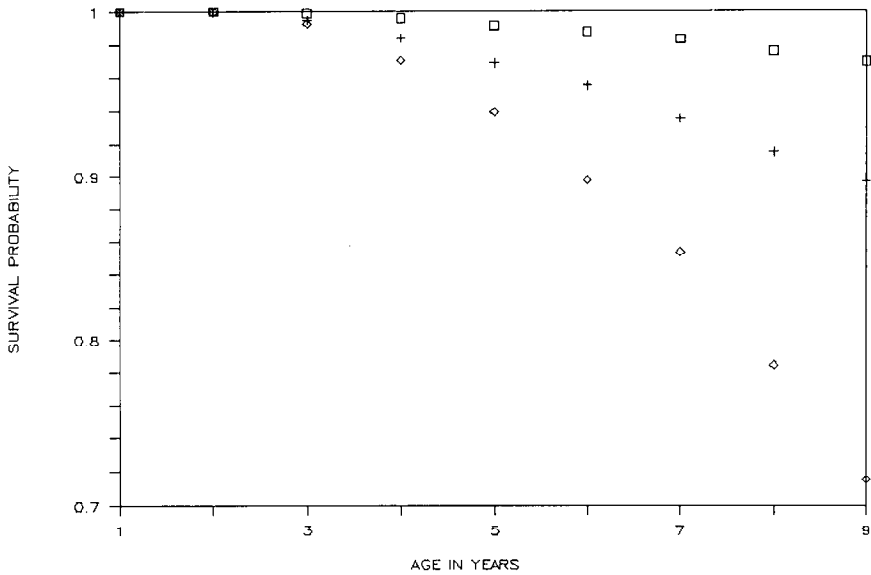


Figure 1. Survival probabilities for mortgages originated in the Western states in 1980 by LTV:  
 Key: □ = LTV < 81%; + = 81% ≤ LTV ≤ 90%; ◇ = LTV ≥ 91%.

loan loss experience across regions. Table 3 indicates the residuals in the estimation of default probabilities using the hazard models. The standard deviation of the residuals is presented together with the correlation of residuals across regions. These data are presented separately for each LTV category.

Table 4 presents similar information for the loan loss rates as a fraction of initial loan size. The entries in table 4 were computed by multiplying the residuals in the estimation of default probabilities by the average loss rates computed separately for the 780 combinations of region, age, LTV, and year of origin. The correlations reported in the tables are rather striking. Holding LTV and age constant, there is a negative correlation between default rates in the Northeastern and the Western regions for the safest loans (LTV ≤ 80 percent). A negative correlation also exists between the Northeastern and Southeastern regions for the riskiest loans.

When the loan loss rates are considered, the correlations are again negative for several comparisons across geographical regions, for example between losses in the Northeast and the Northcentral regions. The pattern of covariances in these returns suggests that portfolio risk can be reduced by geographical diversification, holding other determinants of credit risk constant.

Table 3  
Residuals from hazard estimates of default by region and loan-to-value ratio.

Standard deviation of residual ( $\times 10^2$ )	Correlation of residuals in estimating default probabilities						
	Northeast	Northcentral	Southeast	Southwest	West	U.S.	
<i>LTV</i> $\leq$ 80%							
NE	0.027	1.000	0.375	0.790	0.438	-0.354	-0.180
NC	0.132		1.000	0.689	0.852	0.070	0.549
SE	0.067			1.000	0.650	-0.377	-0.031
SW	0.215				1.000	0.182	0.609
W	0.030					1.000	0.832
U.S.	0.072						1.000
81% < <i>LTV</i> < 90%							
NE	0.057	1.000	0.233	-0.104	0.627	0.236	0.288
NC	0.380		1.000	0.102	0.789	0.744	0.956
SE	0.215			1.000	0.581	-0.495	-0.150
SW	0.734				1.000	0.344	0.756
W	0.094					1.000	0.854
U.S.	0.248						1.000
<i>LTV</i> > 90%							
NE	0.081	1.000	0.651	-0.078	0.606	0.253	0.664
NC	0.722		1.000	0.524	0.814	0.759	0.934
SE	0.149			1.000	0.444	0.788	0.569
SW	1.908				1.000	0.547	0.937
W	0.348					1.000	0.752
U.S.	0.959						1.000

### 3.2. Capital requirements

As discussed above, the recently proposed risk-based capital requirements mandated, among other things, that 3 percent of the unpaid balance of mortgages with LTV less than 80 percent or with insurance be held as a buffer against risk. Almost all of the mortgages in our sample fall into this category.<sup>7</sup> We take this proposal as the 'revealed preference' of regulators and develop a model to calculate relative capital requirements, using the statistical results reported above.

#### 3.2.1. Pricing credit risk

We abstract from issues of transactions costs and taxes, and we focus on pure credit costs (alternatively, we assume that these other costs are nonstochastic and that the lender charges the appropriate fees for them). The

<sup>7</sup>Some of the high LTV loans in this sample did not have commercial insurance; instead the seller retained part of the loan as coinsurance. About 10 percent of the defaults with LTV > 0.8 in our sample fell into this category.

Table 4  
Residuals from predicted and actual loan loss rates by region and loan-to-value ratio.

Standard deviation of residual (loss rates in percent)	Correlation of residuals in estimating loan rates				
	Northeast	Northcentral	Southeast	Southwest	West
<i>LTV</i> ≤ 80%					
NE 0.007	1.000	-0.306	-0.005	-0.043	0.466
NC 0.086		1.000	-0.685	0.900	0.375
SE 0.036			1.000	0.889	0.343
SW 0.135				1.000	0.458
W 0.004					1.000
81% ≤ <i>LTV</i> ≤ 90%					
NE 0.021	1.000	0.441	0.501	0.190	0.391
NC 0.188		1.000	0.405	0.823	0.808
SE 0.041			1.000	0.571	0.863
SW 0.327				1.000	0.482
W 0.022					1.000
<i>LTV</i> > 90%					
NE 0.026	1.000	0.467	0.061	0.528	0.262
NC 0.325		1.000	0.709	0.860	0.813
SE 0.040			1.000	0.499	0.751
SW 1.019				1.000	0.630
W 0.104					1.000

lender who holds a portfolio of mortgages also receives fee income from taking on credit risk. This income, as well as required capital, provides some cushion against bankruptcy. We assume that the credit guarantee fee, *G*, received by lenders is equal to the marginal cost of default losses. More precisely, the lender charges an annual fee such that the expected present value of receipts from *G* equals the expected present value of losses. That is, *G* must be such that

$$\sum_{t=1}^N \frac{G}{(1+r)^t} UPB_t = \sum_{t=1}^N \frac{E(L_t)}{(1+r)^t} UPB_t, \tag{2}$$

where *UPB<sub>t</sub>* is the unpaid balance on the pool, *r*, is the appropriate discount rate, *N* is the term of the mortgages in the pool, and *E(L<sub>t</sub>)* is the expected loss per dollar of *UPB* at time *t*. To simplify computations, we assume that *r* and *UPB* are nonstochastic, so that *L* is the only random variable. *E(L<sub>t</sub>)* can be calculated directly from the hazard model, eq. (1), for mortgages with varying *LTV* in different regions with loss varying by *LTV*, region, age, and year of origin. *UPB* changes largely due to mortgage prepayments. We

assume that prepayments follow the commonly used rule of thumb that also approximates prepayment rates, on average, during the period.<sup>8</sup>

We estimated the  $\lambda_{ij}$  matrices for 13 ages ( $j$ ) and 3 LTV categories ( $i$ ), separately for each of the five regions. The results for the United States as a whole are reproduced<sup>9</sup> in table A.1 in the appendix. Their patterns reveal an increase in hazard up to the 9th or 10th year followed by a decline in hazard with age. We use the  $\lambda$ 's calculated for each region, LTV class, and age, as well as the coefficients reported in table 2, to compute 'fair prices' (i.e. prices yielding a net present value of zero) for credit risk by LTV category. We average across regions and compute prices for the 'representative' origination year during the period. That is, we assume that it is not possible to price credit risk separately by region,<sup>10</sup> and that variations in default risk by origin year are not known ex ante. At an interest rate of 10 percent, we calculate the expected loss rates as a fraction of the initial balance and, using the prepayment assumption, we compute the fee,  $G$ , which yields the required expected present value. Panel A in table 5 summarizes the calculations.

We should note two things about the premiums. First, they are rather low (the fee for loans with LTV below 80 percent is only about two basis points). Second, they vary sharply with LTV (ranging up to 22 basis points for the riskiest loans). That they are low may be, in part, a reflection of the fact that loans originated early in the period benefited from abnormal inflation, and were probably safer than can generally be expected.<sup>11</sup>

### 3.2.2. Determining capital adequacy

We take the approach to capital adequacy indicated above, and apply it to representative pools of loans. The model is a simple difference equation. The pools begin with some capital invested by the lenders; the stock of invested capital is reduced when default costs exceed fee receipts. The pool is bankrupt when it runs out of cash (that is, the pool can neither borrow nor pay dividends).

In period  $t$ , the cash flow ( $CF$ ) is

<sup>8</sup>We assume that the prepayment rate is initially zero and rises linearly over time so that after 2 years, it is at an annual rate of 13 percent. Thirteen percent was chosen so as to make the interest sensitivity of mortgages equal to its historical level. It is roughly the historical average and is a variant on the so-called PSA rule of thumb.

<sup>9</sup>These matrices, the baseline hazards, are estimated by the Kaplan–Meier method for the hazard models reported in table 2.  $\lambda$  matrices estimated separately for each of the regions are available from the authors.

<sup>10</sup>In fact, Congressional wrath would probably prevent price discrimination by region even if ex ante documentation of geographic variation in risk were incontrovertible. Alternatively, each region's experience might be characterized as a separate draw (but with a lot of serial correlation) from a national distribution that does not differ by region.

<sup>11</sup>However, these losses are not much different from those calculated from Freddie Mac loans purchased after 1983, a period of low inflation.

Table 5  
Credit risk fees, standard deviations and capital requirements by loan-to-value ratio and level of diversification.

	Loan-to-value ratio		
	<81%	81-90%	>90%
A. Fees for credit risk <sup>a</sup>	2.2	7.2	22.0
B. Standard deviation			
Portfolio by type of diversification:			
Actual experience weights	2.0	6.8	21.7
Equal shares	4.6	10.8	28.7
Optimal weights	0.4	1.8	2.8
C. Capital requirement <sup>b</sup>			
Undiversified: Baseline portfolio	0.4	0.9	3.0 <sup>c</sup>
Diversified by: Actual experience weights	0.1	0.3	1.0
Equal shares	0.1	0.3	1.1
Optimal weights	0.0	0.1	0.1

<sup>a</sup>Annual fees, as a fraction of unpaid balances, in basis points.

<sup>b</sup>Initial capital required for each portfolio, as percent of loan balances.

<sup>c</sup>Benchmark capital requirement for undiversified mortgages issued in 1980.

$$CF_t = rK_t + G_t - L_t = K_t - K_{t-1}, \quad (3)$$

where  $K_t$  is the capital on hand at the beginning of the period and  $r$  is the interest rate. Again  $G_t$  is the fee for credit risk received, and  $L_t$  represents aggregate losses from default.  $CF_t$  is thus the change in capital. The initial level of capital,  $K_0$ , is the required level, such that the probability that  $K_t$  is zero is less than  $\rho$ .

The only stochastic variable is  $L_t$ , which is the loss rate (as estimated from the hazard models) together with conditional losses times the unpaid balance ( $UPB_t$ ) at the beginning of the period.  $UPB$  declines over time, again because some mortgages are prepaid. The annual premium,  $G$ , is calculated as described in eq. (2).

Given an initial level of capital,  $K_0$ , the variance of loss rates (which will change with diversification), the expected default losses and credit risk premium, we can estimate the probability that  $K_t$  is positive for 10 consecutive years. This estimate is obtained by Monte Carlo techniques. We run simulations of (3) and use the fraction that survive 10 years as an estimate of  $\rho$ . Using the same techniques, we can work backwards and find the value of  $K_0$  that makes this probability, say, 0.96.

Our strategy is to find the  $\rho$  that validates the 3 percent rule. The model permits an analysis of capital requirements in terms of three sources of risk: errors made in forecasting patterns of default probabilities by region, LTV category, and origination year. If authorities were capable of varying capital

requirements along all three dimensions, then capital requirements would arise only from the stochastic disturbances in eq. (1).

These features are captured in the model and its calibration. We interpret the revealed preference of regulators (the FHLBB/OTS proposal) as requiring 3 percent of capital to be withheld to cushion against the worst case of defaults – loans in the riskiest (LTV > 90) category originated in 1980 in a representative but unspecified region, but with no regional diversification. We estimate the probability of bankruptcy in this case by Monte Carlo simulation. We then compute the capital requirements sufficient to yield the same value of  $\rho$  in other circumstances.

The benefits of diversification across regions come from two sources: the covariance matrix of disturbances across regions (see table 4), and the unequal variances of disturbances across regions. A region with a higher variance of disturbances will have a more-than-proportionately higher bankruptcy rate than a region with a lower variance. Hence, even if disturbances were perfectly correlated across regions, a diversified portfolio would require less capital if variances were unequal across regions. The ‘excess capital’ in regions with low variance could be used to protect against risks in high variance regions.

We consider portfolios with returns equal to the national average but with different variances. The variance ( $V$ ) of a portfolio is

$$V = wCw', \quad (4)$$

where  $C$  is the variance–covariance matrix of disturbances and  $w$  is the vector of regional weights. The optimal portfolio is the set of shares that minimizes (4) subject to the constraint that all weights are positive and that they sum to one.

### 3.3. Results

Calibration of the model yields the result that if institutions in the five regions all held 3 percent capital, all had 91–95 percent LTV loans, and were exposed to the worst vintage (1980), then the average institution would have a 0.04 probability of failure over the next decade. The failures would be concentrated in the Northcentral and Southwestern regions, but the average would be 0.04. Hence we use  $\rho = 0.96$  as the criterion for capital adequacy. Compared with this baseline, the model indicates that a 0.9 percent rule is adequate to keep the national failure rate the same for LTVs between 81 and 90 percent. For the safest mortgages with LTVs below 81 percent, only 0.4 percent capital is required. Panel C of table 5 reports these results as the baseline capital requirements.

Part B of table 5 also presents the standard deviations of disturbances for

Table 6  
Capital requirements when credit risk is not priced separately by  
loan-to-value ratio.

	Loan-to-value ratio		
	< 81%	81-90%	> 90%
Portfolio			
Undiversified: Baseline	0.2	0.9	4.3
Diversified: Actual experience	0.0	0.5	2.2
Equal shares	0.0	0.5	2.2

the diversified portfolios discussed above. Note that it is always the case that higher LTV portfolios have higher standard deviations. Part C of table 5 presents the implied capital requirements for these portfolios.

The capital requirements for portfolios weighted by actual loan originations and those weighted equally are similar. In contrast, the optimally weighted portfolio reduces risk, and hence capital requirements, substantially. The optimal weights vary by LTV, but in each case the weights are extreme, requiring little or no investment in the Southeast, Southwest, or the Northcentral region.<sup>12</sup>

Table 6 presents analogous results when credit risk is not priced by LTV category. In these simulations, prices are fixed at 7.2 basis points (the weighted average for all LTV classes). Not surprisingly, capital requirements decline for low LTVs and rise substantially for riskier portfolios. Current industry practice is that credit risk is not priced by LTV, and these results indicate that there are substantial losses from mispricing. The mispricing leads to substantially higher capital requirements for high LTV loans.

## 6. Conclusions and comments

This paper presents a model of defaults, fees, and capital adequacy in which empirical evidence on default rates is used to calculate premiums for different levels of risk. Default rates and their variations, together with premium fees, are then used to calculate capital requirements using a consistent model.

The results indicate clearly that fixed rate, 30 year mortgages on owner-occupied properties with low loan-to-value ratios or with insurance are not very risky. For low LTV loans, 3 percent is clearly a high capital requirement. The experience with loans originated between 1976 and 1980 suggests that premiums of 2-22 basis points would cover that risk, on average.

<sup>12</sup>For LTVs below 81 percent, the unconstrained weights were 140 percent for the West, -40 percent in the East, and zero elsewhere. In the calculations reported in table 5, the weights were set at 100 percent for the West and zero elsewhere.

This analysis does not address other types of mortgages, such as adjustable rate mortgages and multifamily (or, more generally, investor-owned) mortgages. Presumably, these mortgages have greater default risks than those we have analyzed. Note, however, that an increase in average default losses would not affect our conclusions at all if it left the variances and covariances of returns unchanged and if it were incorporated into the fee charged for credit risk.

In this analysis we have avoided the question of whether this recent time period was truly 'representative' and whether there is 'enough' variance in the model by calibrating the volatility of residuals so that the proposed 3 percent capital requirement is 'just right' for LTVs above 90 percent originated in the worst year (1980), and that were not diversified nationally.

Under these and other conditions reported in the paper, diversification clearly matters. The standard deviation of a nationally diversified portfolio is about one-third of an undiversified portfolio, and its estimated capital requirements are between one-third (table 5) and one-half (table 6) of the requirement for an undiversified portfolio. The capital requirement for the optimal portfolio is less by about 80–90 percent. Our results concerning the benefits of geographical diversification are certainly conservative. In making the comparisons, we assume that mortgage pools are diversified within regions; this need not be true for S&Ls. We also ignore diversification across LTVs. The default rates of mortgages of differing LTVs are not perfectly correlated; thus there is some diversification benefit to holding all three LTV types.

## Appendix

Table A.1

Baseline hazards ( $\lambda_{ij}$ ), for the United States as a whole, by age and LTV ratio ( $\times 100$ ).

Age	Loan-to-value ratio		
	<81%	81–90%	>90%
1	0.00	0.02	0.02
2	0.02	0.07	0.11
3	0.06	0.30	0.39
4	0.12	0.44	0.67
5	0.18	0.56	0.90
6	0.14	0.53	1.08
7	0.14	0.57	1.21
8	0.19	0.68	1.83
9	0.28	0.88	2.79
10	0.22	0.85	2.79
11	0.09	0.71	1.84
12	0.08	0.20	0.51
13	0.06	0.12	0.11

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