

## **Loan Loss Severity and Optimal Mortgage Default**

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*This paper tests the contingent claims model of mortgage default in its ruthless or frictionless form. The principal tests of the model are based on an unconventional source of data, namely, loan loss severities on defaulted mortgages. The frictionless model has well-defined predictions about loss severities which we test in detail. The data analyzed include a random sample of all mortgages originated during the period 1975-90 and purchased by Freddie Mac, as well as the loss severities on all mortgages purchased by Freddie Mac which defaulted during the period. The frictionless model does not do well in these tests.*

Over the past few years, there has been increased attention to the pricing of credit risk in the mortgage market (e.g., Cunningham and Hendershott 1984; Kau et al. 1986, 1992; Hendershott and Van Order 1987) and to empirical models of default (e.g., Campbell and Dietrich 1983; Foster and Van Order 1985; Cooperstein, Redburn and Myers 1991; Quigley and Van Order 1992). For the most part, these models focus on the contingent claims approach to default, which treats default as the exercise of a put option.

The importance of homeowner equity, which is a measure of the extent to which the option is "in-the-money," in virtually all studies of default confirms that the options approach is a fruitful way to analyze default. Kau et al. (1992) simulate default frequencies in a frictionless model (i.e., one with no transaction costs) and argue that these frequencies are not much different from those actually observed. They also argue that the introduction of transaction costs into their simulation implies implausibly low default rates. This leads them to conclude that the frictionless model does a good job of explaining default behavior. These are, however, weak tests of qualitative properties. One can introduce different types of transaction costs that are fully consistent with the data on observed default propensities [see Quigley and Van Order (1992)].

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In this paper, we analyze this issue using a completely different body of data: loan loss severities on defaulted mortgages. The severity of loan losses can be interpreted as a measure of the extent to which the option is in-the-money when it is exercised. The frictionless option model has well-defined predictions about loss severities, which we test in some detail. The frictionless model does not do very well in these tests.

The data analyzed in this paper include individual loans originated during the period 1975 through 1991 and purchased by Freddie Mac. We use these data and Freddie Mac's data on loan losses to analyze the loss severities for some 10,000 mortgages which defaulted during the period.

### **Optimal Exercise**

Rational borrowers in a perfectly competitive market will exercise options when they can thereby increase their wealth. Absent either transaction costs or reputation costs (which reduce credit ratings), wealth can be increased by defaulting when the market value of the mortgage equals or exceeds the value of the house. Similarly, by prepaying when market value exceeds par, borrowers can increase wealth by refinancing. Note that the value of the mortgage exceeds the present value of the remaining payment stream because the mortgage claim includes both the options to prepay and also to default at some subsequent date. Thus, even if the market value of the house is less than the present value (discounted at risk-free rates) of future mortgage payments (i.e., the default option is in-the-money), it may not be optimal to exercise the default option.

The problem of determining when to exercise an option requires specifying the underlying state variables and parameters that determine the price of any contingent claim and then deducing the rule for exercise that maximizes borrower wealth. For residential mortgages, the key state variables are interest rates and house values. The value of a mortgage,  $M(c, \mathbf{i}, t, V, T)$ , depends upon the coupon rate,  $c$ , a vector of relevant interest rates,  $\mathbf{i}$ , property value,  $V$ , the age of the mortgage,  $t$ , the remaining time to maturity,  $T$  and various parameters. A standard arbitrage argument is sufficient to derive an equilibrium condition for  $M$  (a second-order partial differential equation), specifying that the expected return plus capital gains must equal the risk-free rate of return plus a risk adjustment. This condition applies to any claim that is contingent on the

underlying state variables. It implies that the value of the mortgage equals the risk-adjusted expected present value of its net cash flows.<sup>1</sup>

To simplify matters and to isolate the default option, assume that interest rates are non-stochastic (so that the only source of risk is house price volatility), and there is no prepayment option. Assume that house price changes are continuous, with an instantaneous mean,  $\mu$  (which need not be constant) and a standard deviation,  $\sigma$ . Let  $\rho$  be the imputed rent payout ("dividend") rate. The arbitrage model implies that the value of the mortgage  $M$  satisfies

$$\left(\frac{1}{2}M^2\sigma^2\right)\left(\frac{\partial^2 M}{\partial V^2}\right) + V(i - \rho)\left(\frac{\partial M}{\partial V}\right) + \left(\frac{\partial M}{\partial t}\right) + C = \mathbf{i}M, \quad (1)$$

where  $\mathbf{i}$  is the interest rate and  $C$  is the coupon payment on the mortgage (which depends on the coupon rate,  $c$ ).

This follows almost directly from the analysis of Black and Scholes (1973). Stochastic house prices are assumed to follow

$$\begin{aligned} dV &= \mu(V,t) dt + \sigma(V,t) dz \\ &= \mu V dt + \sigma V dz, \end{aligned} \quad (2)$$

where  $z$  is a normally distributed error term. Arbitrage-free equilibrium requires that the expected (instantaneous) return,  $\theta M$ , on holding any security,  $M$ , the value of which is contingent on  $V$ , equal the risk-free return plus a risk adjustment based on the market price of house price risk

$$\theta M = \mathbf{i}M + \lambda\sigma\left(\frac{\partial M}{\partial V}\right), \quad (3)$$

where  $\lambda$  is the market price of house-price risk.<sup>2</sup> Equation (3) applies to the house itself, so that in equilibrium the expected return,  $\phi V$ , on holding a house must equal the risk-free return plus the risk adjustment or

$$\phi V = \mathbf{i}V + \lambda\sigma. \quad (4)$$

<sup>1</sup> See Cox, Ingersol and Ross (1985), lemma 4.

<sup>2</sup> For a general derivation of equation (3) and the representation of  $\lambda$  see Brennan and Schwartz (1985).

This expected return is, however, nothing other than the flow of services from the asset, at rate  $\rho$ , and the expected capital gain

$$\phi V = \rho V + \mu V. \quad (5)$$

Finally, the expected return on the mortgage, from Ito's lemma, is given by

$$\theta M = C + \left( \frac{\partial M}{\partial t} \right) + \mu V \left( \frac{\partial M}{\partial V} \right) + \left( \frac{1}{2} \right) \sigma^2 \left( \frac{\partial^2 M}{\partial V^2} \right). \quad (6)$$

Equating (4) and (5), substituting into (3) and equating (3) and (6) yield equation (1). Equation (1) simply states that the expected return (coupon plus expected, risk-adjusted capital gains) must equal the risk-free rate.

Note that the expected appreciation rate  $\mu$  of the house does not appear in equation (1), nor does the price of risk,  $\lambda$ , for holding the asset.<sup>3</sup> This is a quite general result; if the underlying state variables are traded assets, then arbitrage leads to a risk-neutral interpretation of the price of a contingent claim on an asset relative to the price of that asset. The value of the option is the expected present value of the outcome, where prices are projected to grow at a mean rate of  $i - \rho$  (and variance  $\sigma^2 t$ ) and are discounted at the risk-free rate. This is equivalent to assuming risk neutrality [See Smith (1976) for a discussion and Cox, Ingersoll and Ross (1985) for a proof.]

An infinite number of functions satisfy equation (1), which reflects the infinite number of ways that coupon plus capital gain can equal the required expected return. By incorporating the optimal call or put strategies, the function appropriate for a particular mortgage can be determined. Optimal exercise strategies are determined by wealth maximization.

If there are no costs to default other than losing the house, the optimal default "strategy" is characterized simply by the house value at time  $t$ ,  $V_t^*$ , at which default takes place. The optimal  $V_t^*$  minimizes the value of the mortgage (this maximizes the borrower's net worth), subject to the condition that  $V_t^*$  equal the value of the remaining balance when the option is exercised.

<sup>3</sup> This is because equations (4) and (5) imply that  $(i - \rho)V = \mu V - \lambda \sigma$ .

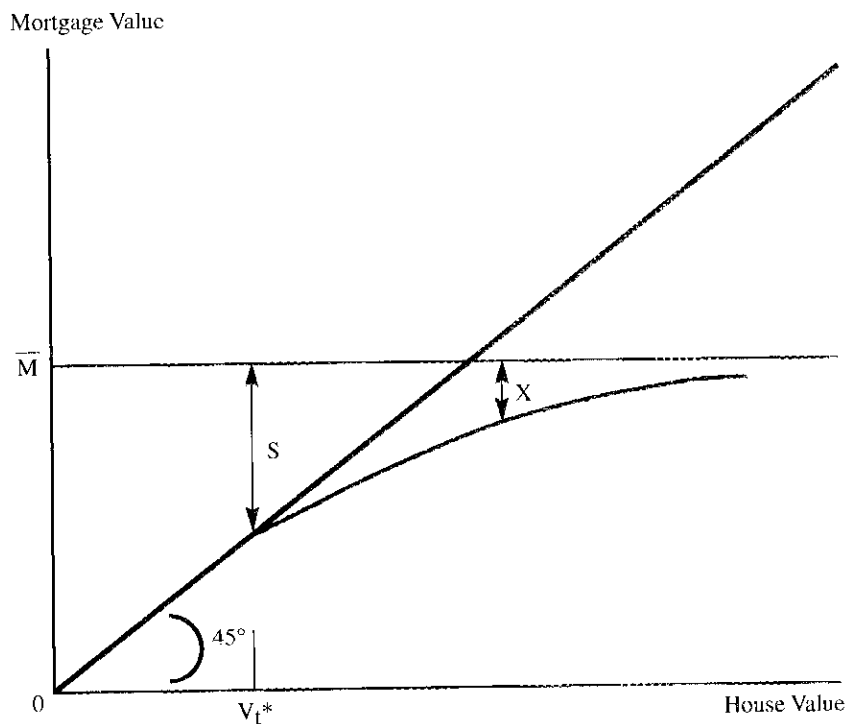
Figure 1, adapted from Quigley and Van Order (1992), illustrates the optimal strategy. This strategy is represented by the lowest curve (i.e., the one that minimizes  $M$ ) which satisfies equation (1) and is not above the  $45^\circ$  line (where the remaining balance equals the value of the house). If the solution is an interior one, it is represented by the tangency depicted in the figure. The curve must also be below the horizontal line  $\bar{M}$ , which gives the value of a riskless mortgage. The curve approaches  $\bar{M}$  asymptotically as  $V$  increases. The tangency determines  $V_i^*$ , the default strategy. The entire curve gives the market relationship between mortgage values and house prices. The distance  $X$  is the value of the default option, the premium for insurance that a competitive mortgage insurer would charge. At  $V_i^*$ , the distance  $S (= X)$  represents the extent to which the option must be in-the-money before default. Note, however, that this distance is also the severity of the loss by the lender or mortgage insurer (absent transaction costs) from selling the house immediately after foreclosure.

The virtue of the contingent claim model is its simplicity. The default option is exercised at  $V_i^*$ , which depends only on the variables in equation (1) and on the boundary and tangency conditions, which depend on the same variables. The equilibrium condition has the property that both the mean price change of any traded asset as well as the risk premium are irrelevant in pricing the option or in exercising it. Thus, circumstances under which default occurs depend only on  $\rho$ ,  $\sigma$ ,  $c$ ,  $T$ ,  $M$  and  $V$ ; they are independent of the original house price, expected price appreciation, the original loan-to-value ratio (LTV), the historic path of prices and the market price of risk.

The logic of the model is relatively straightforward. The "cost" of exercising the option now, even if the option is in-the-money, is the inability to exercise it later on. Things that cause the option value to be high will cause the borrower to delay exercising it until it is further in-the-money, causing severity ( $S$  in Figure 1) to be higher. In particular, the longer the time to maturity, the lower the coupon rate relative to current interest rates, the higher the rental rate and the higher is price volatility, the more valuable is the option. We cannot measure the last two with our data set. However, from the data that we have we can test four propositions about loss severities each of which is consistent with the ruthless or frictionless model.

- (1) *Ceteris paribus*, severity should be independent of initial LTV. However, high LTV loans almost always have insurance if they are purchased by Freddie Mac. The cost of insurance increases the effective coupon rate to the borrower for high LTV loans.

Figure 1 ■ Optimal Default



but not the mortgage coupon rate measured in financial data. Thus for this data set, the frictionless model predicts that severity should fall as initial LTV increases.

- (2) *Ceteris paribus*, severity should be the same in regions with high default frequencies as in regions with low frequencies and should be the same for loans originated in “good” years (e.g., those followed by house price inflation) as in “bad” years.
- (3) Severity should decrease with the age of the mortgage. This is because older loans have a shorter time to maturity, and therefore there is less value to keeping the option alive. The option will be exercised when it is less far into-the-money.
- (4) Severity should decrease as coupon rate minus the current interest rate increases. The higher the coupon, the less value there is in keeping the option alive, and thus the sooner it will be exercised.

### Data and Results

Table 1 tabulates loss severity for all defaults on single-family, owner-occupied, non-condominium loans purchased by Freddie Mac within 1 year of their origination during the period 1975–90.<sup>4</sup> Loss severity, gross of any insurance payments, is measured by the difference between the mortgage balance and the value of the house for defaulted loans. It excludes all transaction costs and foregone interest. House values are measured in two ways. The first is based on an appraisal at the time a defaulted property is acquired by Freddie Mac. The second is the actual sale price when (about a year later) the house is sold from the Freddie Mac inventory. Neither is a perfect measure of the extent to which the option was in-the-money when the borrower chose to default. Nonetheless, there is no reason to believe there is a systematic bias by LTV, coupon rate, interest rate or age.

Panel A presents loss severity as a fraction of loan balance for the loans described above that defaulted from 1975–90. The first column indicates the losses based on appraisal data at acquisition (“Loss I”), while the second column uses eventual selling prices (“Loss II”). On average, actual losses consistently exceeded appraised losses by about 8% to 10%. In both columns, however, there is a strong effect of LTV. High LTV loans have much higher severity rates. This is not consistent with the first prediction of the ruthless model.

Panel B of the table presents similar calculations for defaults in Texas during the same time period. The LTV effect remains, though it appears to be much smaller. However, the losses in Texas are substantially higher. This is not consistent with the second prediction of the frictionless model.<sup>5</sup>

Of course, these static comparisons do not hold other things constant. For instance, the table does not control for the age of the loan. *Ceteris paribus*, high LTV loans will have negative equity at younger ages than low LTV loans. Because younger loans have larger option values than older ones, severity should be lower. To control for this and for other factors, we report regressions relating individual severity rates on defaulted loans

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<sup>4</sup> Seasoned loans acquired by Freddie Mac were eliminated from these comparisons.

<sup>5</sup> Institutional differences, such as homestead provisions and state laws requiring delays in enforcing eviction, may cause average loss rates to vary among states. In general, however, Texas provides fewer protections against eviction than any other state. Thus, based on institutional differences alone, Texas loss rates should be lower than elsewhere. See Claurctie and Herzog (1989).

**Table 1** ■ Loss severity by loan-to-value (LTV) as a percent of mortgage balance, 1975-1990.

Original	Loss I	Loss II
A. All Loans		
LTV 51-60%	-7.0%	2.0%
LTV 61-70%	-0.3	8.7
LTV 71-75%	0.9	10.0
LTV 76-80%	2.1	11.7
LTV 81-90%	5.2	15.1
LTV > 91%	14.8	24.3
B. Texas Loans		
LTV 51-60%	-1.5%	6.9%
LTV 61-70%	13.1	21.6
LTV 71-75%	13.8	21.7
LTV 76-80%	16.7	24.5
LTV 81-90%	18.2	26.8
LTV > 91%	23.0	32.0

Average losses on all defaulted loans, 1975-1990, excluding defaults on seasoned loans purchased by Freddie Mac. Loss I is computed as mortgage balance minus appraised value at acquisition. Loss II is computed as mortgage balance minus actual sales price at time of sale.

Source: Freddie Mac.

to LTV categories, a dummy variable for Texas loans, dummy variables for origination years, the age of the mortgage at the time of default and coupon rate minus current mortgage rates.

#### *A Crude Test*

Table 2 presents these regressions. In columns 1 and 2, the dependent variable is the loss rate estimated at the time of property acquisition by Freddie Mac. The results in columns 3 and 4 rely upon loss severity rates computed using the actual sale prices when Freddie Mac sells the property. The predictions for LTV are soundly rejected, as is the prediction

**Table 2 ■** Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
LTV 51-60% (Dummy)	7.64 (0.97)	7.73 (0.98)	11.86 (1.58)	11.68 (1.56)
LTV 61-70% (Dummy)	11.82 (1.66)	11.92 (1.67)	16.04 (2.36)	15.85 (2.34)
LTV 71-75% (Dummy)	17.65 (2.49)	17.76 (2.51)	19.02 (2.83)	18.81 (2.79)
LTV 76-80% (Dummy)	23.24 (3.36)	23.35 (3.38)	21.75 (3.31)	21.54 (3.28)
LTV 81-90% (Dummy)	27.94 (4.05)	28.03 (4.07)	26.94 (4.11)	26.76 (4.09)
LTV > 90% (Dummy)	34.71 (5.04)	34.82 (5.06)	34.70 (5.30)	34.47 (5.26)
Age of Mortgage (Thousand Days)	-2.38 (7.80)	-2.40 (7.90)	-2.94 (10.13)	-2.90 (10.03)
Coupon Minus Current Rate (Percent)	-24.90 (0.76)		48.15 (1.55)	
Texas (Dummy)	11.74 (13.77)	11.83 (13.99)	7.47 (9.21)	7.31 (9.09)
Intercept	-25.11 (3.64)	-25.15 (3.65)	-19.27 (2.94)	-19.18 (2.93)
R-Squared	0.07	0.07	0.06	0.06
F-Value	74.2	83.4	63.5	71.2
Observations	9457	9457	9457	9457

Loss I is measured by appraised value minus mortgage balance. Loss II is measured by value at time of sale minus mortgage balance. *t* ratios are shown in parentheses.

for Texas loans. Indeed, the results are quite similar to those reported in Table 1. The age of the loan has a negative effect, as is predicted by the theory, and coupon minus rate varies in sign and significance. The results for age and coupon appear to be fragile. In Appendix Table A.1, we add dummy variables for origination years 1975 through 1989 and quadratic terms for coupon minus rate and age. This causes the signs of age and coupon to be ambiguous, but results for LTV and Texas appear to be robust.

As can be seen from in Appendix Table A.1, the dummy variables for origination year are highly significant. Losses were much lower on mortgages originated in the 1970s when inflation was high than they were subsequently when inflation was low. This also contradicts the second proposition.

#### *A More Precise Test*

Of course, the observations on loss severities analyzed in Tables 1, 2 and A.1 are hardly a random sample of the millions of mortgages purchased by Freddie Mac during the 1975–90 time period. In particular, the “rule” for selecting observations for the analysis of loss severities is the fact of default by the borrower. The contingent claims model predicts that the probability of default is itself a function of homeowner equity. Homeowner equity is a function of initial LTV and the subsequent course of house prices, which vary by geographical region and time period. Thus, the selectivity of the sample implies that the coefficients may be biased measures of the unconditional loss severities in the population.

Consistent estimates of the behavioral parameters can be obtained by including the Mills ratio of the selection rule as an additional variable in the regression [see Heckman (1976)]. The Mills ratio ( $f/[1 - F]$ , where  $f$  is the probability density function and  $F$  is the cumulative density function) can be computed directly from the hazard of default for each observation in the analysis sample.

Appendix B presents estimates of the default hazard for a random sample of 277,199 observations on mortgages purchased by Freddie Mac during the 1976–90 period (2,036 of these mortgages actually defaulted). The coefficients of this proportional hazards model, together with the estimates underlying baseline hazard (estimated by the method of Meier and Kaplan, also presented in the Appendix), permit us to compute the probability of default and hence the Mills ratio for each of the 10,000 observations in the analysis sample.

Table 3 presents the same specification as reported previously, this time including the Mills ratio as an additional explanatory variable. The dependent variable is the loss rate calculated by each of the two methods described previously.

In each of the regressions reported, the Mills ratio is significant and is quite large in magnitude. For example, a coefficient of about 500 in the regressions suggests that an observation with a predicted probability of default of one half percent will have a loss rate, on average, about two percentage points higher than an observation with a predicted probability

**Table 3 ■** Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
LTV 51-60% (Dummy)	7.75 (0.99)	7.69 (0.98)	12.03 (1.61)	11.63 (1.56)
LTV 61-70% (Dummy)	11.80 (1.65)	11.73 (1.65)	16.00 (2.37)	15.58 (2.30)
LTV 71-75% (Dummy)	17.37 (2.46)	17.30 (2.45)	18.62 (2.78)	18.19 (2.71)
LTV 76-80% (Dummy)	22.88 (3.32)	22.82 (3.31)	21.22 (3.24)	20.83 (3.18)
LTV 81-90% (Dummy)	26.96 (3.92)	26.92 (3.91)	25.51 (3.91)	25.28 (3.87)
LTV > 90% (Dummy)	32.46 (4.71)	32.43 (4.71)	31.42 (4.80)	31.27 (4.78)
Age of Mortgage (Thousand Days)	-1.77 (5.44)	-1.77 (5.44)	-2.05 (6.64)	-2.06 (6.66)
Coupon Minus Current Rate (Percent)	18.75 (0.56)		111.81 (3.51)	
Texas (Dummy)	10.65 (12.16)	10.62 (12.15)	5.88 (7.07)	5.69 (6.86)
Mills Ratio	305.84 (5.31)	298.00 (5.34)	446.00 (8.17)	399.20 (7.54)
Intercept	-26.18 (3.80)	-26.12 (3.79)	-20.84 (3.19)	-20.48 (3.13)
R-Squared	0.07	0.07	0.06	0.06
F Value	69.8	77.5	64.3	72.5
Observations	9457	9457	9457	9457

Loss I is measured by appraised value minus mortgage balance. Loss II is measured by value at time of sale minus mortgage balance. *t* ratios are shown in parentheses.

of default of one-tenth of 1%.<sup>6</sup> The historical average for Freddie Mac is around 0.2%.

Despite this, however, the signs and levels of significance of the other variables are unchanged, and the magnitudes are similar. Again, the Appendix (Appendix Table A.2) presents the same specification, but with the addition of quadratic terms and a set of dummy variables indicating year of origination. Once again, the dummy variables for origination year are highly significant. The pattern of coefficients does not arise from the selectivity of observations into the sample of loan losses.

The central finding remains. Each of the four hypotheses is either rejected by the data or is at best shaky.

### Conclusions

The frictionless model predicts that, holding term and coupon constant, there is an optimal amount of “in-the-moneyness” that will induce borrowers to default, and that amount is independent of initial LTV. Our results show a strong relationship to LTV, and even after econometric adjustments, the results by LTV are not much different from those given in Table 1, which depicts the raw data by LTV.

Indeed, the data appear to be consistent with the hypothesis that people wait until the values of their houses have dropped by 20% (for Loss I) or 30% (for Loss II), and then they default. This rule of thumb is hardly consistent with a sophisticated notion of wealth maximizing behavior.

Suppose, however, that transaction costs and liquidity are important. Specifically, suppose that transaction or reputation costs are such that [as in Kau et al. (1992)] people seldom exercise their options “ruthlessly.” Assume instead that people “get into trouble,” e.g., by losing a job, and have only enough liquidity to last a specified but random period, (e.g., averaging one year), and at the end of that period they must either sell the house or default. Under these circumstances, they may default if the option is just barely in-the-money.

At the end of the period, all those “in trouble” will have experienced housing price declines of approximately the same percent, which means that loans with higher initial LTVs will have larger loss severities. This model is consistent with our data and the empirical results presented in this paper. In particular, these complications are consistent with the pro-

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<sup>6</sup> i.e.,  $500 \left( \frac{0.005}{0.995} - \frac{0.001}{0.999} \right) \approx 2$ .

nounced effects of LTV on loss severities with the importance of the dummy variables for origination year and for loans made in Texas during this period.

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**Table A.1** ■ Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
LTV 51–60% (Dummy)	8.30 (1.11)	8.23 (1.10)	12.54 (1.72)	12.57 (1.73)
LTV 61–70% (Dummy)	13.90 (2.05)	13.83 (2.04)	17.69 (2.67)	17.71 (2.68)
LTV 71–75% (Dummy)	20.18 (3.00)	20.12 (2.99)	21.18 (3.23)	21.19 (3.24)
LTV 76–80% (Dummy)	26.98 (4.11)	27.14 (4.14)	24.76 (3.87)	25.18 (3.94)
LTV 81–90% (Dummy)	33.41 (5.10)	33.56 (5.13)	30.89 (4.83)	31.31 (4.91)
LTV > 90% (Dummy)	38.70 (5.91)	38.78 (5.93)	37.49 (5.87)	37.76 (5.93)
Age of Mortgage (Thousand Days)	7.88 (17.16)	13.10 (9.89)	3.58 (7.98)	12.79 (9.93)
Age of Mortgage Squared ( $\times 1E7$ )		-- 10.37 (4.20)		-- 18.42 (7.66)
Coupon Minus Current Rate (Percent)	160.45 (4.59)	180.30 (4.78)	171.91 (5.04)	220.20 (5.99)
Coupon Minus Current Rate Squared ( $\times 1E-2$ )		4.30 (0.30)		20.82 (1.51)
Texas (Dummy)	6.55 (7.71)	6.66 (7.84)	4.85 (5.85)	5.04 (6.10)
Origination Year 75 (Dummy)	-79.44 (3.44)	-79.71 (3.46)	-58.57 (2.60)	-58.95 (2.63)
Origination Year 76 (Dummy)	-74.99 (3.34)	-76.99 (3.43)	-47.96 (2.19)	-51.48 (2.35)
Origination Year 77 (Dummy)	-59.62 (2.68)	-- 62.87 (2.83)	-39.26 (1.81)	44.99 (2.08)
Origination Year 78 (Dummy)	-50.02 (2.26)	-- 54.06 (2.44)	-- 29.55 (1.37)	-36.70 (1.70)
Origination Year 79 (Dummy)	-- 41.84 (1.89)	-- 46.28 (2.09)	-21.52 (1.00)	29.48 (1.37)
Origination Year 80 (Dummy)	-38.77 (1.75)	-42.83 (1.93)	22.51 (1.04)	-29.84 (1.38)
Origination Year 81 (Dummy)	-31.13 (1.41)	-34.31 (1.55)	-19.02 (0.88)	-25.04 (1.16)

**Table A.1** ■ Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
Origination Year 82 (Dummy)	-18.94 (0.86)	-22.17 (1.00)	-7.46 (0.35)	-13.58 (0.63)
Origination Year 83 (Dummy)	-19.23 (0.87)	-22.76 (1.03)	-7.21 (0.33)	-13.43 (0.62)
Origination Year 84 (Dummy)	-13.95 (0.63)	-17.39 (0.79)	-4.79 (0.22)	-10.95 (0.51)
Origination Year 85 (Dummy)	-14.19 (0.64)	-17.25 (0.78)	-7.14 (0.33)	-12.59 (0.58)
Origination Year 86 (Dummy)	-12.17 (0.55)	-15.01 (0.68)	-6.93 (0.32)	-11.98 (0.56)
Origination Year 87 (Dummy)	-12.73 (0.57)	-15.21 (0.69)	-7.74 (0.36)	-12.15 (0.56)
Origination Year 88 (Dummy)	-11.51 (0.51)	-13.48 (0.60)	-5.16 (0.24)	-8.67 (0.40)
Origination Year 89 (Dummy)	-1.52 (0.07)	-2.58 (0.11)	8.07 (0.36)	6.11 (0.27)
Intercept	-17.23 (0.75)	-18.66 (0.81)	-17.35 (0.77)	-20.06 (0.89)
R-Squared	0.16	0.16	0.10	0.11
F Value	74.6	69.7	46.5	45.6
Observations	9457	9457	9457	9457

Loss I is measured by appraised value minus mortgage balance. Loss II is measured by value at time of sale minus mortgage balance. *t* ratios are shown in parentheses.

**Table A.2** ■ Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
LTV 51-60% (Dummy)	8.38 (1.12)	8.32 (1.11)	12.67 (1.74)	12.71 (1.75)
LTV 61-70% (Dummy)	13.72 (2.02)	13.71 (2.02)	17.39 (2.63)	17.50 (2.66)

**Table A.2** ■ Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
LTV 71–75% (Dummy)	19.78 (2.94)	19.81 (2.95)	20.48 (3.13)	20.66 (3.16)
LTV 76–80% (Dummy)	26.44 (4.03)	26.69 (4.07)	23.82 (3.73)	24.41 (3.83)
LTV 81–90% (Dummy)	32.44 (4.95)	32.77 (5.00)	29.21 (4.58)	29.95 (4.70)
LTV > 90% (Dummy)	36.92 (5.63)	37.33 (5.69)	34.41 (5.38)	35.27 (5.53)
Age of Mortgage (Thousand Days)	8.09 (17.48)	12.63 (9.49)	3.93 (8.72)	12.02 (9.28)
Age of Mortgage Squared ( $\times 1E7$ )		-9.15 (3.65)		16.32 (6.69)
Coupon Minus Current Rate (Percent)	166.04 (4.75)	183.69 (4.87)	181.57 (5.33)	226.01 (6.15)
Coupon Minus Current Rate Squared ( $\times 1E-2$ )		4.99 (0.35)		21.99 (1.59)
Texas (Dummy)	5.62 (6.32)	5.89 (6.60)	3.25 (3.74)	3.72 (4.29)
Origination Year 75 (Dummy)	-78.73 (3.41)	-79.09 (3.43)	-57.34 (2.55)	-57.89 (2.58)
Origination Year 76 (Dummy)	-74.69 (3.33)	-76.51 (3.41)	-47.46 (2.17)	-50.66 (2.32)
Origination Year 77 (Dummy)	-59.18 (2.66)	-62.12 (2.80)	-38.50 (1.78)	-43.72 (2.02)
Origination Year 78 (Dummy)	-49.68 (2.24)	-53.30 (2.40)	-28.96 (1.34)	-35.41 (1.64)
Origination Year 79 (Dummy)	-41.89 (1.89)	-45.80 (2.07)	-21.60 (1.00)	-28.65 (1.33)
Origination Year 80 (Dummy)	-39.55 (1.79)	43.00 (1.94)	-23.86 (1.11)	-30.13 (1.40)
Origination Year 81 (Dummy)	-32.57 (1.47)	-35.14 (1.59)	-21.51 (1.00)	-26.47 (1.23)
Origination Year 82 (Dummy)	-21.11 (0.95)	-23.59 (1.07)	-11.21 (0.52)	-16.01 (0.74)
Origination Year 83 (Dummy)	-20.97 (0.95)	-23.75 (1.07)	-10.21 (0.47)	-15.14 (0.70)

**Table A.2** ■ Regression models of actual loss severity in percent.

	Loss I		Loss II	
	1	2	3	4
Origination Year 84 (Dummy)	-15.49 (0.70)	-18.24 (0.82)	-7.45 (0.35)	-12.41 (0.58)
Origination Year 85 (Dummy)	-14.84 (0.67)	-17.42 (0.79)	-8.27 (0.38)	-12.89 (0.60)
Origination Year 86 (Dummy)	-11.88 (0.54)	-14.44 (0.65)	-6.43 (0.30)	-10.99 (0.51)
Origination Year 87 (Dummy)	-12.42 (0.56)	-14.67 (0.66)	-7.20 (0.33)	-11.22 (0.52)
Origination Year 88 (Dummy)	-11.18 (0.50)	-12.98 (0.58)	-4.59 (0.21)	-7.81 (0.36)
Origination Year 89 (Dummy)	-1.54 (0.07)	-2.48 (0.11)	8.04 (0.36)	6.30 (0.28)
Intercept	-16.91 (0.73)	-18.24 (0.79)	-16.79 (0.75)	-19.34 (0.86)
R-Squared	0.16	0.16	0.11	0.11
F Value	72.2	67.4	46.4	44.9
Observations	9457	9457	9457	9457

Loss I is measured by appraised value minus mortgage balance. Loss II is measured by value at time of sale minus mortgage balance. *t* ratios are shown in parentheses.

### Appendix B

As discussed in Quigley and Van Order (1992), the contingent claims model implies that the hazard of default is a function of homeowner equity. In particular, some level of negative equity is associated with a much higher probability of exercising the put option. Table B.1 presents parameter estimates of a proportional hazards model estimated from a random sample of 277,199 mortgages purchased by Freddie Mac during the 1976–1990 period. The model includes the determinants of individual equity: the initial LTV, region and the origination year. The parameters of the hazard model are highly significant, and the parameters are broadly

consistent with Quigley and Van Order (1992). The table does not include the baseline hazards which were estimated by the method of Kaplan and Meier [see Kalbfleisch and Prentice (1980)]. These parameters, including the baseline, are used to estimate the Mills ratio for each of the observations in the sample analyzed in the text.

**Table B.1 ■ Hazard model of default.**

Origination Year (Dummy)	1976	0.11 (2.6)
	1977	0.25 (0.7)
	1978	1.31 (4.1)
	1979	2.24 (7.0)
	1980	3.22 (3.8)
	1981	3.70 (11.6)
	1982	3.50 (10.3)
	1983	3.30 (10.0)
	1984	3.13 (9.9)
	1985	2.76 (8.4)
	1986	1.74 (5.3)
	1987	1.83 (3.9)
	1988	1.83 (5.4)
	1989	2.29 (6.7)
	1990	2.64 (7.8)

**Table B.1 ■** Hazard model of default.

Region (Dummy)	NC	--0.12 (1.7)
	NE	--0.31 (4.0)
	SE	--0.03 (0.4)
	SW	0.91 (1.5)
LTV 61--70%		1.17 (6.5)
LTV 71--75%		1.78 (10.2)
LTV 76--80%		1.90 (12.7)
LTV 81--90%		2.57 (16.7)
LTV > 90%		3.16 (20.4)

*t* ratios in parentheses.