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STATISTICAL ANALYSIS OF SALES DATA
TO VERIFY APPRAISAL INFORMATION

BY

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Statistical Analysis of Sales Data to Verify Appraisal Information*

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Statistical models of real estate price determination can be an important tool in managing the appraisal function. The accuracy of these models depends crucially on the inclusion of all relevant information as well as on the correct specification of the price function.

This paper proposes a hybrid model of the hedonic price relationship, combining information on single sales and multiple sales of properties. The paper also indicates the utility of such a model in the verification of appraisal data.

Introduction

Computer-based statistical models of real estate markets have been employed for more than twenty years in the assessment, appraisal, and verification of market prices and in the forecasting of future real estate prices. Compared with less systematic methods of appraisal and assessment, computer-based models provide a number of clear advantages. A more widespread diffusion of computer models into the markets for private and public sector real estate appraisals depends crucially on exploiting the advantages of such systematic methods while remaining conscious of their inherent limitations.

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In this paper, we consider the role of computerized appraisal techniques. We discuss some of the practical advantages and disadvantages of statistical valuation models and compare a standard regression model of real estate prices with a weighted repeat sales model. We then suggest a hybrid approach that incorporates the desirable properties of each method. We also present and compare empirical estimates of a simple model of real estate prices using these three methods. Finally, we describe one practical use of these models in the verification of private appraisals.

Advantages and Disadvantages of Statistical Appraisal Methods

Computer-assisted real estate models offer a number of advantages over more traditional methods. Perhaps most important, these methods provide rigorous controls for the varying characteristics of properties. An important factor distinguishing the real estate market from most other economic markets is the heterogeneity of the product. This heterogeneity, or product differentiation, is particularly evident in the residential sector, where the sale of a property involves a simultaneous transaction in the markets for land, for structures, for neighborhoods, for locations, and for access to the local public goods produced by governments. These latter markets, for neighborhoods, locations, and access to local public goods such as schools and police protection, include many trades in valuable attributes that are not even produced by landlords or developers.

The computer-assisted models that have been developed to analyze real estate markets provide well-known methods to control for the joint influence of these important characteristics on the observed transaction prices of properties. In this way, they allow the investigation of price trends over time or the comparison of properties standardized with respect to all but one physical or locational characteristic.

A second advantage of computer-based models is that they permit the partition of the observed variation in real estate prices into individual determining factors. From the viewpoint of local officials and government planners, for example, statistical models can be used to isolate and forecast the price implications of public policies affecting property values, such as restrictions on the availability of land for development or improvements in accessibility to jobs and amenities. In the private sector, the same partitioning can be used to isolate and estimate the market return to investment in particular attributes or types of properties or in particular locations.

Finally, computerized appraisal models are of considerable potential value in guiding investors in their portfolio allocations. For example, all sophisticated theories of individual investment conclude that the allocation of an investment portfolio across securities should depend on the mean
return and the variability of returns (or risk) to the various kinds of securities. For most investments, historical data permit the returns to be calculated easily from a lively spot market in the commodity in question. In real estate markets, however, trades are relatively infrequent, and the markets are rather thin. Thus the estimation of an investment return series for real estate depends crucially on methods of imputation—that is, on methods that use an observed sample of transactions to infer the market value of the unsold stock of property. Virtually all investment return series in real estate are calculated by some form of imputation of market prices based upon appraisal techniques. These appraisal techniques are ideally suited to computer-based statistical modeling to represent expected property values and price over time.

On the other hand, it must be recognized that computer-assisted real estate valuation models have rather well-defined limitations. Accurate and reliable statistical analysis depends crucially on identification of the variables relevant to the analysis, as well as on correct specification of the form in which these variables should be included. If one or more factors influencing property values are excluded from the statistical model, the conclusions may be systematically incorrect. Similarly, even if all relevant factors are included, if they are specified improperly, the results may be misleading.

Of course, if extraneous information is included in a model it will not ordinarily affect the results, and statistical methods can help eliminate extraneous information. But statistical analysis alone cannot establish whether the relevant information has been captured in any model, nor can statistical results alone verify that the form in which this information has been used is “correct” in any theoretical sense. The models and the methods themselves are no substitute for more fundamental inquiry into the operation of the real estate market. Stated another way, guidance in the choice of empirical techniques, variables, and sources of data, as well as the functional form for relationships, depends on a satisfactory theory of the real estate market.

More generally, if consumers in the market differ by the cost of acquiring information, and suppliers recognize these differences, then there may exist an equilibrium distribution of market prices rather than a single market price for comparable real estate. The function of the appraiser, then, is to extract the “signal” from the “noise” associated with each observation on real estate transactions. Presumably, as information is revealed over time to participants in the market, the equilibrium price distribution will tend to converge to a single market price. However, the appraisal techniques used may inhibit or encourage this convergence of equilibrium prices. From this perspective, computer-assisted appraisal techniques may represent the most rigorous systematic method of assimilating information on market behavior.
The technological changes of the past decade have accentuated both the advantages and the disadvantages of computerized appraisal. Reductions in the cost of computation and in the availability of computers have made it extremely inexpensive to gain access to basic information on real estate sales and real estate characteristics. The wide availability of statistical software packages has made it relatively simple for those with only rudimentary training in quantitative methods to estimate sophisticated statistical pricing models. However, the very simplicity of estimation has meant that decision makers can easily be inundated with superficially similar statistical models of appraisal. Models with similar statistical properties, but using different information (or simply using information differently), may yield quite different forecasts, especially for time periods or properties outside the range of the sample used in estimation itself. Only rigorously specified models using all available information can be expected to generate similar conclusions and forecasts.

Full Use of Available Information in Real Estate Appraisal Models

Given the inherent inability of computer appraisal techniques themselves to establish the “correct” representation of the determinants of real estate value, and given the tendency of such models to generate incorrect conclusions when relevant information is excluded, it is imperative that statistical methods begin by using all available information in estimating market prices. Unfortunately, most current models of real estate prices typically fail to use fully the available information on property values, especially when the same properties are sold several times.

A Hybrid Model

Consider, for example, a regression model of the relationship between the value $V_t$ of a property at time $t$, its physical and locational characteristics, $X$, and some representation of time:

$$V_t = f(X, t). \quad (1)$$

Interpretation of this relationship depends crucially on the inclusion of the correct set of property characteristics, $X$, and the correct functional form, $f(\cdot)$, for the regression. Conditional upon these factors, however, the statistical model can be used to forecast the market price for a standardized, or “quality adjusted,” property over time. As an appraisal tool, this standard hedonic regression facilitates an appraisal of the value, $\tilde{V}_t$, of a property with characteristics $\tilde{X}$ at time $\tilde{t}$ through the computation of

$$\tilde{V}_t = f(\tilde{X}, \tilde{t}). \quad (2)$$

Even when the correct set of property characteristics and the correct
functional form are employed, however, the standard method may fail to use fully the available information. The value of any specific property that was sold twice during a given time period presumably varies only with respect to time, since the physical and locational characteristics of that property would not have changed. But the standard model expressed in equation 1 treats multiple sales of the same property in the same way as multiple individual sales of very similar (but not identical) properties. In short, with multiple sales, the standard model does not use completely the information on property characteristics.

Because of this problem, it has been suggested that the characteristics of each property be completely standardized with reference only to themselves, by confining the statistical analysis to properties that have been sold more than once during a given time period (Bailey, Muth & Nourse 1963):

$$V_i/V_t = g(t,\hat{t}).$$

(3)

This formulation, the weighted repeat sales method, avoids the problems of the standard model by relating a change in the selling price of a property between time $\hat{t}$ and time $t$ only to the timing of the two transactions, or perhaps to the time interval $(t - \hat{t})$ between sales. Price indices for single-family houses have been computed using this technique by Mark and Goldberg (1984), Palmquist (1980), and, more recently, by Case and Shiller (1987).

Although this latter approach avoids the difficulty of specifying and measuring the various quality characteristics of real properties, it does so at considerable cost. By confining the analysis to properties sold more than once, it can be extremely wasteful of sales information. In any market run, the fraction of properties sold more than once is bound to be small. The estimation strategy implicit in equation (3) simply ignores all information on the sales prices and the characteristics of single transactions.

Also, the weighted repeat sales technique is inappropriate when any of the characteristics of a property have been changed between sales dates. More important, success in the identification of properties with changed physical or locational characteristics entails the exclusion of still more information on market values of such properties. It is a curious research strategy indeed that excludes so much available information from the analysis.

Table 1 summarizes one measure of the sacrifice in information made by the recent studies using the weighted repeat sales method. The table indicates that despite the long time periods included (from 14.5 years to 22 years), only a small fraction of the observed sales in the seven housing markets analyzed were repeat transactions. Thus, by confining the statistical analysis to repeat sales, almost two-thirds of Palmquist's sample
TABLE 1

Samples of Sales and Repeat Sales in Studies Using Weighted Repeat Sales Methods for Single-Family Housing

<table>
<thead>
<tr>
<th>Study and housing markets analyzed</th>
<th>Analysis time period</th>
<th>Total sales</th>
<th>Repeat sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Palmquist (1980)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King County, Washington</td>
<td>14.5 yrs</td>
<td>4,785</td>
<td>1,613</td>
</tr>
<tr>
<td>Mark and Goldberg (1984)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraser, Vancouver</td>
<td>22.0 yrs</td>
<td>4,376</td>
<td>1,695</td>
</tr>
<tr>
<td>Kerrisdale, Vancouver</td>
<td>22.0 yrs</td>
<td>1,398</td>
<td>660</td>
</tr>
<tr>
<td>Case and Shiller (1987)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atlanta, Georgia</td>
<td>16.5 yrs</td>
<td>221,876</td>
<td>8,945</td>
</tr>
<tr>
<td>Chicago, Illinois</td>
<td>16.5 yrs</td>
<td>397,183</td>
<td>15,530</td>
</tr>
<tr>
<td>Dallas, Texas</td>
<td>16.5 yrs</td>
<td>211,638</td>
<td>6,669</td>
</tr>
<tr>
<td>San Francisco, California</td>
<td>16.5 yrs</td>
<td>121,909</td>
<td>8,066</td>
</tr>
</tbody>
</table>

of single-family housing transactions was eliminated; in Case and Shiller’s analysis, more than 97 percent of the observed transactions were ignored.

Consider now an alternative research strategy that combines the desirable features of equations (1) and (3): it uses information on all transactions, whether single or multiple transactions; it capitalizes on the additional information available when multiple transactions are observed on any property. We call this the hybrid method.

To focus on the essentials, consider a very simplified model of real estate values in which initial property values, $V_0$, vary with the qualitative and quantitative aspects of properties, $X$, according to a simple exponential relation:

$$\log V_0 = \log A + a \log X, \quad (4)$$

where $A$ and $a$ are parameters. Moreover, the value of a property at a given time, $V_t$, varies continuously over time $t$:

$$\log V_t = \log V_0 + bt, \quad (5)$$

or, equivalently,

$$\log V_t = \log A + a \log X + bt, \quad (6)$$

where $b$ is a parameter. This simple model implies that if we observe a single transaction at time $t$, the selling price of the property can be esti-
mated from (6), which describes the standard model. Suppose, however, we observe two sales of a property, at \( t \) and \( i \), whose characteristics are unchanged during the interval from \( i \) to \( t \). From (5), the estimated selling price is

\[
\log V_t = \log V_i + b(t - i),
\]

which describes the weighted repeat sales model.

Finally, suppose we observe two sales of a property at \( t \) and \( i \), but, in this case, suppose the characteristics of the property are changed from \( X \) to \( X^* \) at \( t^* \), \( i < t^* < t \). In this case it can be shown that

\[
\log V_t = \log V_i + a \log(X^*/X) + b(t - i),
\]

where the property characteristics included in the set \( X \) are continuously measured.

Equations 5, 6, 7, and 8 provide alternative methods for estimating the parameters of the appraisal model for three kinds of samples. Consistent estimates of the parameters can be obtained from any of the three samples, at least as long as the samples are random.

Clearly, if information is available for two or more of these samples, as in the studies reported in table 1, then all the information should be used to estimate the parameters of the model. This is accomplished by estimating the appropriate equations jointly in the model of real estate prices. The estimates themselves may be computed by generalized least squares (see Case and Quigley [1988] for details).

**Empirical Analysis**

The model presented in equations 5, 6, 7, and 8 is estimated from observations on the sales of single detached housing from the Kahala neighborhood of Honolulu, Hawaii, during the period October 1980 to October 1987. The neighborhood consists of about 1,100 residential parcels and is bounded by Kahala Beach, the Waialae Golf Course, and the Kalanianaole Highway. The sample for this analysis consists of every sale in this neighborhood during the seven-year period: 418 residential transactions involving 310 separate properties.

The 1980s were a period of rapidly rising prices in Hawaii, from an already high base; this neighborhood is no exception. The median sale price of these dwellings was $372,000. In 1980, the average transaction was for $361,000. In 1987, unadjusted sales prices averaged $845,000. More detail on the average characteristics of the sampled dwellings is reported in table 2.

Table 3 summarizes the regression estimates based upon this body of data. It represents estimates based upon the standard method (equation 6), the weighted repeat sales method (equation 7), and the hybrid method.
TABLE 2
Summary Data and Regression Results
Standard deviation and t ratio of estimates in parentheses

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean value</th>
<th>Standard model</th>
<th>Weighted repeat sales model</th>
<th>Hybrid model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept: $A$</td>
<td>-</td>
<td>7.479</td>
<td>7.532</td>
<td></td>
</tr>
<tr>
<td>Land area: $a_1$ (square feet)</td>
<td>12,186</td>
<td>0.378</td>
<td>0.301</td>
<td></td>
</tr>
<tr>
<td>Distance to shore: $a_2$ (feet)</td>
<td>1,664</td>
<td>-0.166</td>
<td>-0.166</td>
<td></td>
</tr>
<tr>
<td>Living area: $a_3$ (square feet)</td>
<td>2,257</td>
<td>0.370</td>
<td>0.455</td>
<td></td>
</tr>
<tr>
<td>Other covered area: $a_4$ (square feet)</td>
<td>557</td>
<td>-0.013</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>Age: $a_5$ (years)</td>
<td>25.4</td>
<td>-0.057</td>
<td>-0.072</td>
<td></td>
</tr>
<tr>
<td>Bedrooms: $a_6$ (number)</td>
<td>3.57</td>
<td>-0.123</td>
<td>-0.082</td>
<td></td>
</tr>
<tr>
<td>Bathrooms: $a_7$ (number)</td>
<td>2.63</td>
<td>0.085</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>Fee simple: $a_8$ (dummy)</td>
<td>0.69</td>
<td>0.288</td>
<td>0.332</td>
<td></td>
</tr>
<tr>
<td>Swimming pool: $a_9$ (dummy)</td>
<td>0.51</td>
<td>0.059</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td>Time: $b$ (thousands of days)</td>
<td>1.51</td>
<td>0.217</td>
<td>-0.136</td>
<td>0.069</td>
</tr>
</tbody>
</table>

*Statistically significant at the 0.05 level.
**Statistically significant at the 0.01 level.

(equations 6, 7, and 8). For completeness, it also presents estimates of equation 8 for those properties whose characteristics have been changed, as well as estimates of equation 6 for the subsample of properties sold only once.

As the table indicates, an analysis based on the weighted repeat sales method is extremely wasteful of the information contained in this sample. Of the 418 transactions observed during the seven-year period, only 108 involved multiple transactions on the same property, and of these only 47 involved multiple transactions on properties with unchanged characteristics. An analysis based on the weighted repeat sales method uses only 11 percent of the data.

The table also reports the coefficients estimated for the time parameter $b$, as well as its standard error, and the explained variation in sales prices implied by each model. As compared to the results of the so-called standard model, the results of the weighted repeat sales model are suspect—despite the high proportion of the variation in sales prices accounted for by the model, the standard error associated with $b$ is much larger and the estimated coefficient itself, contrary to expectations, is negative.
TABLE 3
Estimates of Real Estate Price Model
All House Sales in Kahala, Honolulu
October 1980—October 1987

<table>
<thead>
<tr>
<th>Specification and sample</th>
<th>Number of observations</th>
<th>Estimated coefficient, $b$</th>
<th>Standard error of coefficient, $b$</th>
<th>Explained variation in model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sales</td>
<td>418</td>
<td>0.217</td>
<td>0.022</td>
<td>0.804</td>
</tr>
<tr>
<td>Single sales only</td>
<td>310</td>
<td>0.179</td>
<td>0.022</td>
<td>0.796</td>
</tr>
<tr>
<td>Multiple sales models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted repeat sales</td>
<td>47</td>
<td>−0.136</td>
<td>0.097</td>
<td>0.955</td>
</tr>
<tr>
<td>Sales of changed properties</td>
<td>61</td>
<td>0.059</td>
<td>0.099</td>
<td>0.897</td>
</tr>
<tr>
<td>Hybrid model</td>
<td>418</td>
<td>0.069</td>
<td>0.017</td>
<td>0.887</td>
</tr>
</tbody>
</table>

*Each regression also includes a set of $X$ variables which measure land area, distance to the shore, living area, other covered area, and the age of structure, as well as the number of bedrooms and bathrooms, and binary variables for the presence of a swimming pool and for the conveyance of title in fee simple. Complete regression results for rows 1, 3, and 5 are reported in table 2.

In comparison with either of these models, the hybrid model proposed in this section performs better as measured by statistical significance, and better than the standard model as measured by explained variation. Since the hybrid model uses the information recorded for all sales and capitalizes on the additional information provided by repeat sales, this result is to be expected. For this sample, however, the gains in statistical significance appear to be quite large.

Practical Application to Verification of Appraisals

The statistical model developed in the last section could have widespread applicability in the management and evaluation of real property appraisals. The statistical model could be used as a complement to traditional appraisal methods or to verify the accuracy and reliability of traditional appraisals.

To illustrate this, figure 1 shows a graphic representation of some of the results reported in table 2. Specifically, it presents the 80 percent confidence interval associated with forecasts of the market price over time of a property with average physical and locational characteristics as estimated using the standard and the hybrid models. For reasons already noted, the confidence interval is considerably narrower for the hybrid model.

Suppose, for illustration, that a lender has estimated the hybrid model indicated by the solid line and the associated confidence interval, and that
an appraiser has subsequently presented an estimate of market value for
some specific property with given values of the characteristics \( x \) included
in the model. Let the symbol \( X \) on the diagram represent the appraised
value.

The distance \( d \) on the diagram is a measure *ex ante* of the confidence
of the lender in the report submitted by the appraiser. The larger the
distance \( d \), the less likely it is that both the statistical model and the
appraisal are valid. As the diagram is drawn, this can be given an explicit
probabilistic interpretation: if the model is correct, then the probability
that the actual value of the property is equal to or greater than the ap-
praised value \( X \) is only 0.2, corresponding to the distance \( d \).

This interpretation suggests that the probability associated with an ap-
praisal can be used as a management tool by lenders, insurers, public
agencies, or other participants in real estate markets. If, conditional upon
the statistical model, the value reported by the appraiser is relatively un-
likely, then additional resources should be devoted to verification of the
appraisal value. The particular decision rule adopted by a manager will
depend on the cost of verifying an appraisal and the cost associated with
appraisal error.

Presumably the costs associated with appraisal error are asymmetric.
In the housing market, for example, an appraisal that is "too low" may
result in lost business for a lender or insurer, whereas an appraisal that
is "too high" will generally increase the default risk to the lender or in-
surer. Similarly, a low appraisal would result in lost property tax revenues for a public agency, but a high appraisal might be considered confiscatory.

Clearly it can be of significant benefit for a lender or other participant in the real estate market to evaluate the accuracy and reliability of independent property appraisals. The evidence based on this sample suggests that the hybrid model could permit substantial improvements over other more conventional approaches in the evaluation of appraised property values.

References


