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NEUTRAL PROPERTY TAXATION

By

Richard Arnott

UNIVERSITY OF CALIFORNIA, BERKELEY

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Richard Arnott
Boston College

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Abstract

A major difficulty in implementing land/site value taxation is imputing the land value of built-on sites. The literature has focussed on two alternatives. The first, *residual site value,* measures post-development site value as property value less structure value, measured as depreciated construction costs. Residual site value would be relatively easy to estimate, but residual site value taxation is distortionary, discouraging density. The second, *raw site value*, measures post-development site value as "what the land would be worth were there no building on the site (though in fact there is)". Raw site value taxation is neutral (does not distort the timing and density of development), but the estimation of raw site value would be complex so that assessment would likely be less fair and more arbitrary, contentious, and prone to abuse.

This paper asks the question: Is it not possible to design a *property tax system* (taxation of pre-development land value, post-development structure value, and post-development site value at possibly different rates) that employs the administratively simpler residual definition of post-development site value *and* achieves neutrality? The paper provides an affirmative answer, characterizes the tax rates that achieve neutrality, and briefly discusses issues of practical implementation.

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Department of Economics
Boston College
Chestnut Hill, MA 02167
USA
Tel: (617) 552-3674
Fax: (617) 552-2308
E-mail: richard.arnott@bc.edu

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Neutral Property Taxation

Suppose, for the sake of argument, that a parcel of land has an "intrinsic value" that is unaffected by decisions concerning its current use. A tax on such intrinsic value would be neutral — would not affect decisions concerning its current use. This principle has led many economists through the years to advocate the use of land value (or, synonymously, site value) taxation, and the replacement of the current non-neutral property tax system with a land value tax system.

The obvious difficulty is to come up with a definition of land value that is not only neutral, but also fair, practicable and sensible. One unavoidable problem is that because of the durability and immobility of structures, there is no "market" value for developed land. The value observed in the market for a developed site is its property value, and there is no economically correct way to decompose this value into land value and structure value.

During the 1970's four papers (Shoup (1970), Skouras (1978), Bentick (1979), and Mills (1981)) independently examined arguably the most intuitive decomposition, defining post-development site value as property value minus the depreciated cost of the structure on the site. This definition, here termed *residual site value*,[1] is appealing because it is intuitive and would be relatively easy to implement for tax purposes. The results of these papers can be obtained from a simple model of a developer who owns a unit area of vacant land. He must decide, under perfect foresight, when to construct a durable structure on the site and at what density. A land or property tax system is said to be neutral if its application does not alter the developer's timing or density decisions. What Shoup et al. (who collectively shall be referred to as *the revisionists*) showed was that, in the absence of other taxes, the taxation of residual site value is non-neutral; in particular, it discourages density. This result received widespread attention since it called into question the conventional wisdom concerning the neutrality of land taxation.

Subsequent work (Tideman (1982)) has shown that neutrality *is* achieved when post-development site value is instead defined as "what the site would be worth if there were no structure on it (even though in fact there is)" — here termed *raw site value* — since this value is unaffected by the developer's current decisions. Use of this hypothetical value has the disadvantage, however, that it cannot be simply calculated or estimated on the basis of market observables. The taxation of raw site value — however calculated — would likely therefore be capricious and unfair, encourage corruption, and give rise to extensive and wasteful assessment appeals. Thus, it would appear that the choice of definition of post-development site value for site value taxation purposes entails a tradeoff between deviations from neutrality and administrative costs, broadly interpreted.

This paper asks whether it is not possible to get the best of both worlds — to avoid the tradeoff — with a well-chosen *property* tax system. More specifically, with separate tax rates on pre-development land value, post-development *residual* site value, and structure value, is neutrality achievable? Since there are three objectives — neutrality with respect to development timing, neutrality with respect to development density, and expropriation of a desired fraction of value — and three instruments, a positive answer is plausible. At least for the model employed — which assumes perfect competition, zero rent on vacant land and no uncertainty, among other things — the paper proves that there is indeed a neutral property tax system which employs the residual definition of site value. This positive result provides a basis for optimism in the search for a property tax system that is both practicable and close to neutral. The paper goes on to derive the tax rates that achieve neutrality for the central case where post-development rents grow at a constant rate: Pre-development land value

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[1] This is the definition of post-development site value employed in Hong Kong's land value tax. See Wong (1999).

is untaxed, post-development residual site value is taxed at a rate chosen to meet the revenue requirement, and structure value is *subsidized*.

Section I sets the stage by providing a detailed synthetic review of the literature. Section II presents the analytical results concerning neutral property taxation, and briefly discusses some of the problems that would be encountered in moving from theory to practice. Section III summarizes and concludes.

I. Setting the Stage

To begin, a few words on the terminology employed in the paper are appropriate. A distinction is made between a site value tax system and a property tax system. A site value tax system is characterized by a single tax rate, the same before and after development. The basis for site value taxation prior to development is the market value of the vacant land. After development, however, when there is a durable and immobile structure on the site, there are not separate *market* values for the site and the structure. Site value is then an abstract or hypothetical notion and must be imputed. As we shall see, whether site value taxation is neutral hinges on the definition of post-development site value employed. A property tax system, meanwhile, is characterized by three tax rates: a tax rate on the market value of vacant land which applies prior to development, and separate post-development tax rates on site value and structure value. Thus, according to this terminology, a site value tax system is a special case of a property tax system.

I.1 Synthesis of the Literature on the Taxation of Land

The previous literature has employed a variety of models. The relevant results obtained can be illustrated using extensions of the Arnott-Lewis (1979) partial equilibrium model of the transition of land to urban use.[2]

An atomistic landowner owns a unit area of undeveloped land. He must decide when to develop the land and at what density to build the structure. Once built, the structure is immutable; no depreciation occurs and no redevelopment is possible. He makes his decision under perfect foresight (and hence under no uncertainty).

To start, consider the landowner-developer's problem in the absence of taxation. The following notation is employed:

t	time (t=0 today)
T	development time
K	development density (the capital-land ratio)
Q(K)	structure production function $(Q' > 0,\ Q'' < 0)$
r(t)	rent per unit of structure at time t
i	interest rate
p	price per unit of capital

The structure production function indicates how many units of structure are produced when K units of capital are applied to the unit area of land. For concreteness, one may think of Q as the number of units of rentable floor area per unit area of land (the floor-area ratio), or the number of storeys in the building on the site. The interest rate, the price per unit of capital, and the structure production function are assumed invariant over time to simplify the analysis.

Under the simplifying assumption that land prior to development generates no rent, the developer's problem in the absence of taxation is[3]

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2 Recent contributions to the literature include Turnbull (1988) and McFarlane (1999).

3 It is assumed throughout the paper that infinity paradoxes do not occur, e.g., that $\int_T^\infty r(t)e^{-it}dt$ is finite.

$$\max_{T,K} \quad \Pi(T,K) = \int_T^\infty r(t)Q(K)e^{-it}dt - pKe^{-iT}. \tag{1}$$

The first-order conditions are

$$T: \quad \left(-r(T)Q(K) + ipK\right)e^{-iT} = 0 \tag{2}$$

$$K: \quad \left(\int_T^\infty r(t)Q'(K)e^{-i(t-T)}dt - p\right)e^{-iT} = 0. \tag{3}$$

Eq. (2) states that, K fixed, development time should be such that the marginal benefit from postponing construction one period (the one-period opportunity cost of construction funds) equal the marginal cost (the rent forgone). Eq. (3) states that, T fixed, capital should be added to the land up to the point where the increase in rental revenue due to an extra unit of capital, discounted to development time, equal the cost of the unit of capital. Figure 1 plots (2) and (3) in T-K space[4]. At a local maximum, both (2) and (3) are positively-sloped, and (2) is steeper than (3). To ease notation, since there will be no ambiguity, K will denote either the variable K or the profit- maximizing value of K; ditto for T.

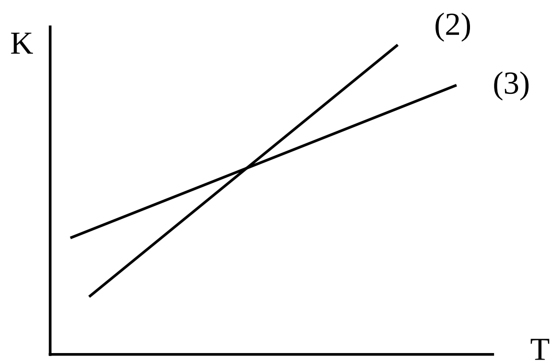

Figure 1: First-order conditions in the Arnott-Lewis model.

[4] The second-order conditions are standard. For the special case where rents grow at a constant, positive rate, a sufficient condition for unique maximum (which is interior) is that the elasticity of substitution between capital and land in the production of structure be less than one.

In terms of Figure 1: $\left(\frac{dK}{dT}\right)_{(2)} = -\frac{\Pi_{TT}}{\Pi_{TK}}, \left(\frac{dK}{dT}\right)_{(3)} = -\frac{\Pi_{KT}}{\Pi_{KK}}, \Pi_{TK} = \Pi_{KT} > 0$ by the concavity of $Q(K)$, so that the second-order conditions $\Pi_{TT} < 0, \Pi_{KK} < 0$, and $\Pi_{TT}\Pi_{KK} - (\Pi_{KT})^2 > 0$ imply that (2) and (3) are both positively-sloped in T-K space, with (2) having the steeper slope.

Property taxes are now introduced into the model. Other forms of taxation — such as income taxation — are assumed away; thus, the property taxes are examined in isolation. Furthermore, no attention is paid to the disposition of tax revenue. This assumption rules out the possibility that the extra tax revenue generated by a tax rate increase is spent on amenity improvements which raise structure rents. A site value or property tax system is said to be *neutral* if it results in the same (T,K) as solve (2) and (3). This definition is standard. Neutrality implies that (in the absence of distortions) the tax system is efficient.

This section examines only site taxation. To proceed with the analysis, additional terminology and notation shall be needed:

n(t) site rent

τ_n tax rate on site rent

V(t) pre-development market value of (vacant) land

P(t) post-development property value

S(t) residual site value

τ tax rate under a residual site value tax system

S(t) raw site value

τ_S tax rate under a raw site value tax system

Prior to development, *site rent* equals the market rent on vacant land, which has been assumed equal to zero. Post-development site rent equals property rent minus amortized construction cost. Thus,

$$n(t) = \begin{cases} 0 & t < T \\ r(t)Q(K) - ipK & t > T. \end{cases} \tag{4}$$

Most of the earlier literature employed static models, and hence failed to distinguish between rents and values. The first economists to use dynamic models, the revisionists, employed the residual site value definition of site value. Pre-development residual site value is the pre-development market value of land. Post-development residual site value equals property value minus depreciated structure value. Here the depreciation rate is assumed to be zero. Accordingly,

$$S(t) = \begin{cases} V(t) & t < T \\ P(t) - pK & t > T. \end{cases} \tag{5}$$

As we shall see, residual site value taxation is distortionary. It has subsequently been recognized (e.g., Tideman (1982, undated), Netzer (1998), and Ladd (1997) who will be referred to collectively as *defenders of the orthodoxy*) that the neutrality of site value taxation can be recovered by employing definitions of site value having the feature that post-development site value is unaffected by the timing and density of development chosen by the market. One such definition of site value is raw site value. Pre-development raw site value — like pre-development residual site value — is the market value of vacant land. Post-development raw site value is what the site would sell for were there no structure on it (even though there in fact is). Thus,

$$S(t) = \begin{cases} V(t) & t < T \\ \Phi(t) & t > T, \end{cases} \tag{6}$$

where $\Phi(t)$ is "what the site would sell for were there no structure on it", an expression for which shall be derived subsequently.

The literature contains five principal results relating to land/site taxation. The following review establishes each result and provides the economic intuition.

- Result 1: A "pure" land value tax — one which is imposed on the "intrinsic" value of the land, independent of the developer's decisions concerning the timing and density of development — is neutral.

Since the tax payable is independent of the developer's decisions, he views such a tax as a lump-sum tax, so it does not affect his decisions.

This is the idea underlying the neutrality of land value taxation. The neutrality result holds however the intrinsic value of the land is calculated (as long as it is independent of the developer's decisions) and whether the tax rate is constant or variable over time.

- Result 2: A linear, time-invariant tax on site *rent* is neutral.

Proof: The developer chooses T and K to maximize the discounted present value of structure rent, less construction costs, less tax payments:

$$\max_{T,K} \int_T^{\infty} r(t)Q(K)e^{-it}dt - pKe^{-iT} - \int_T^{\infty} \tau_n n(t)e^{-it}dt$$

$$= \int_T^{\infty} r(t)Q(K)e^{-it}dt - \int_T^{\infty} ipKe^{-it}dt - \int_T^{\infty} \tau_n\big(r(t)Q(K) - ipK\big)e^{-it}dt \qquad \text{(using (4))}$$

$$= (1-\tau_n)\int_T^{\infty}\big(r(t)Q(K) - ipK\big)e^{-it}dt. \qquad (7)$$

The maximizing choices of T and K are independent of τ_n. ■

In the absence of taxation, the developer chooses T and K to maximize the discounted present value of site rent. With site rent taxation at rate τ_n, the developer chooses T and K to maximize the discounted present value of site rent net of the tax payment. Since the tax payment equals τ_n times site rent, the maximizing T and K are unaffected by the tax. Site rent is analogous to profit, and the neutrality of site rent taxation analogous to the well-known neutrality of a time-invariant tax on pure profit.

Observe that, with durable structures, a site rent tax whose tax rate is time-varying is not in general neutral. Such a tax does not affect the timing first-order condition, but it does distort the density first-order condition. To see this, consider the top storey of a building in the no-tax situation. Suppose in the early years of the building's life, from T to $\hat{t}$, that the top storey loses money (its net rent is negative: $r(t)Q'(K) - ip < 0$), with these losses being exactly offset in discounted terms by profits in later years. Now impose a site rent tax that is set at a positive rate from T to $\hat{t}$ and at a zero rate thereafter. The top storey is subsidized from T to $\hat{t}$ and incurs no tax liability thereafter. This particular time-varying site rent tax would encourage construction at higher density than in the no-tax situation.

- Result 3: A tax on *raw* site value is neutral.

Appendix 1 proves the result for a linear, time-invariant tax on raw site value, but from the intuition given earlier it clearly generalizes.

- Result 4: If structures are perfectly malleable or mobile — so that the developer chooses the *function* K(t) — a linear site value tax is neutral even when the tax rate varies over time.

Proof: To simplify, assume that $Q'(0) = \infty$, so that development occurs at all points in time. Since capital may be regarded as being mobile — rented at ip per unit per unit time — the market value of land is well-defined. Consequently, there is no ambiguity in the definition of site value which is denoted by $\Sigma(t)$:

$$\Sigma(t) = \max_{K(u)} \int_t^\infty r(u)Q(K(u))e^{-i(u-t)}du - \int_t^\infty ipK(u)e^{-i(u-t)}du - \int_t^\infty \tau_\Sigma(u)\Sigma(u)e^{-i(u-t)}du, \quad (8)$$

where $\tau_\Sigma(u)$ is the site value tax rate at time u. Since structures are perfectly malleable, there is no development timing condition. Differentiating (8) w.r.t. t yields

$$\dot{\Sigma}(t) = -r(t)Q(K(t)) + ipK(t) + i\Sigma(t) + \tau_\Sigma(t)\Sigma(t),$$

and solving gives

$$\Sigma(t) = \max_{K(u)} \int_t^\infty \left(r(u)Q(K(u)) - ipK(u)\right)e^{-\int_t^u (i+\tau_\Sigma(u'))du'}du, \quad (9)$$

from which it is evident that linear site value taxation does not affect development density. ■

The intuition is straightforward. Today's site value is essentially independent of today's capital intensity, and future site values completely independent of it. Thus, in deciding on today's capital intensity, the developer views the present value of future site value tax liabilities as a lump sum, and hence his capital intensity decision is unaffected by site value taxation.[5]

Return to the situation where the development decision is completely irreversible — once vacant land is developed at a certain density, it remains at that density forever.

- Result 5: A linear, time-invariant tax on *residual* site value is distortionary.

Proof:

$$S(t) = \begin{cases} \max\limits_{K,T} \int_T^\infty r(u)Q(K)e^{-i(u-t)}du - pKe^{-i(T-t)} - \tau\int_t^\infty S(u)e^{-i(u-t)}dt & t < T \\ \int_t^\infty r(u)Q(K)e^{-i(u-t)}du - pK - \tau\int_t^\infty S(u)e^{-i(u-t)}dt & t > T. \end{cases} \tag{10}$$

Solving S(t) by the now-familiar procedure yields

$$S(t) = \begin{cases} \max\limits_{K,T} \int_T^\infty n(u)e^{-(i+\tau)(u-t)}du & t < T \\ \int_t^\infty n(u)e^{-(i+\tau)(u-t)}du & t > T. \end{cases} \tag{11}$$

The first-order condition with respect to development time is unaffected by the residual site value tax: $n(T) = 0$. The first-order condition with respect to density is, however, distorted. In particular, the tax on residual site value has the effect of increasing the discount rate on site rent from i to $i + \tau$. ■

The marginal cost of postponing development equals the rent forgone, and the marginal benefit from postponing development equals the interest on construction costs plus net tax savings. Since residual site value is the same immediately before and immediately after development and since the tax rate on pre-development residual site value is the same as that on post-development residual site value, the net tax savings from postponing development equal zero, and the timing first-order condition

[5] It is clear that with perfect malleable structures site rent taxation is neutral even with a *time-varying* tax rate.

reduces to what it is in the absence of the residual site value tax.

Two different intuitions for why residual site value taxation distorts the development density first-order condition are now presented. To simplify, consider the normal case in which rents rise monotonically over time. The developer will add storeys to his building up to the point where the discounted net rent from the top storey equals zero, with the negative net rent in earlier years of the building's life just being offset, in discounted terms, by the positive net rent achieved in later years. The residual site value tax raises the discount rate, which puts greater weight on the earlier years when net rent is negative. Thus, holding development time constant, the top storey that just broke even in the absence of the tax loses money when the tax is imposed, implying that the rise in the discount rate caused by the residual site value tax lowers profit-maximizing development density. An alternative explanation is as follows. Start with the situation without the residual site value tax. At development time, the top storey of the building just breaks even. In other words, *at development time* the increment to residual site value from the top storey is zero. Subsequent to development time, the present value of rent from the top storey increases while the present value of amortized construction costs remains constant. Thus, after development time the increment to residual site value from the top storey is positive (and increasing over time). When, therefore, a residual site value tax is imposed, the top storey adds to the building's discounted tax liability. Imposition of the residual site value tax therefore renders the top storey of the building unprofitable. Hence, the residual site value tax discourages density.

The essential difference between raw site value and residual site value taxation should now be apparent. Post-development raw site value is unaffected by the density of development, while in the neighborhood of the optimum post-development residual site value is increasing in the density of development. Thus, imposition of a raw site value tax has no effect on the development density condition, while imposition of a residual site value tax discourages density.

To recapitulate: Result 4 was that with perfectly mobile and malleable structure capital, linear site value taxation is neutral. Results 2, 3, and 5 were derived for the opposite extreme where structure capital is completely immobile and immalleable, but apply as well for intermediate situations where capital can be moved but at a cost and where density can be altered but with adjustment costs. Result 2 was that a linear tax on site rent, at a time-invariant rate, is neutral; result 3 was that raw site value taxation is neutral; and result 5 was that a linear, time-invariant tax on residual site value is non-neutral. Thus, the non-neutrality of a linear, time-invariant tax on residual site value derives from a combination of the immobility and immalleability of structure capital, the taxation of site value rather than site rent, and the particular definition of site value employed.

I.2 Practicability of Alternative Definitions of Site Value

The economists who have written recently on site value taxation can be divided into two camps. Defenders of the orthodoxy and modern Georgists[6] view it almost as an article of faith that site value taxation is neutral. They have therefore objected strongly to assertions that site value taxation is distortionary (Tideman (1982), Netzer (1999)). Their view is not unreasonable. Results 1 and 3 of the previous subsection show that land or site value can be defined so that site value taxation is non-distortionary; raw site value is one such definition. The revisionists, however, employ an alternative definition of site value, residual site value. As with raw site value, pre-development residual site value equals the market value of land. In contrast to the definition of raw site value, however, post-development residual site value is measured as property value minus structure value. Under this definition, site value taxation is indeed distortionary, as was shown in Result 5.

[6] Henry George was an influential, late-nineteenth century Progressive American reformer who argued in favor of a single tax — a confiscatory tax on land rents. Modern Georgists, while not generally adhering to George's view that a

The difference between the two camps therefore derives from differences in the definition of post-development site value, which reflects the absence of separate market values for land and structure on a developed site. In tax policy practice, the choice of definition of post-development site value should be made on pragmatic grounds. Employing the raw site value definition has the advantage that its use results in a site value tax that is neutral. The residual site value definition has the advantage that its computation, though not without contentious aspects, is relatively straightforward.

Vickrey (1970) characteristically understood the economics of site value taxation before anyone else and also characteristically leaned as far as could reasonably be defended towards the theoretically nice policy: "On the whole --- I am inclined to recommend sticking as closely as possible to a theoretically defined [land or site] value" (p.36). But he also acknowledged that " [i]n the end it seems likely that some degree of departure from the goal of strict neutrality will have to be accepted in order to achieve an acceptable degree of administrative feasibility" (p. 29).

Other economists (including myself) would place more weight on administrative feasibility. The further one moves away from a definition of site value based on market observables, the more capricious[7], unfair, and prone to corruption is site value taxation in practice likely to be. Furthermore, the more capricious the tax system, the greater the amount of wasteful litigation. To reduce appeals, assessments for property tax purposes are now routinely based on hedonic price analysis. A site value tax system that defined site value in a way that could not be strongly defended in court, on the basis of market observables, would invite appeals, and for that reason would likely come to be replaced by a system that defined site value on the basis of market observables. Defining site value as residual site value is not ideal in this regard, since imputed post-development structure value, measured by

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confiscatory tax on land values is the *single* tax needed for optimal taxation, subscribe to the view that land value taxation is efficient.

depreciated construction costs, is not simple to estimate. Style obsolescence is hard to measure, and depreciation due to quality deterioration is not only hard to measure but also depends on the level of maintenance chosen which reflects market conditions (Sweeney (1974)). Nevertheless, studies have been undertaken which estimate average rates of depreciation on structures, as captured by age-of-building variables (e.g., Chinloy (1979)), and the results of such studies could be employed to impute post-development structure value. Thus, with hedonic price analysis being employed to estimate post-development property value, residual site value could be imputed using methods that are both straightforward and 'scientific', and therefore readily defensible in court.[8] Hence, on grounds of both administrative costs and fairness, a strong argument can be made for defining site value as residual site value.

Residual site value taxation is, however, distortionary; in particular, it discourages density. A question then arises which, surprisingly, has not been addressed in the literature: Is it possible to design a *property* tax system — defined as linear taxes at different rates that are constant over time on pre-development land value, post-development site value, and post-development structure value — that employs the residual definition of site value and which is neutral or close to neutral? The next section takes up this question.

II. Neutral or Near-neutral Property Taxation

II.1 Analysis and Results

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[7] One can imagine a clerk in the Assessment Department of Small Town, USA, confronted with the task of computing post-development raw site value!

[8] Mills (1998) argues that post-development residual site value would be estimated with an unacceptable degree of error since it is computed as the differences between property value and structure value, each of which, he asserts, would be estimated with considerable error.

The valuation formulae are now derived for the model of the previous section, but with a property tax system instead of a site value tax system. The property tax system is characterized by a linear tax on pre-development land value at rate τ_V, a linear tax on post-development residual site value at rate τ_S, and a linear tax on post-development structure value at rate τ_K, with each tax rate being invariant over time.

Post-development residual site value is

$$S(t) = \int_t^\infty r(u)Q(K)e^{-i(u-t)}du - pK - \tau_S \int_t^\infty S(u)e^{-i(u-t)}du - \tau_K \int_t^\infty pKe^{-i(u-t)}du$$

$$= \int_t^\infty \left[r(u)Q(K) - ipK - \tau_S S(u) - \tau_K pK\right]e^{-i(u-t)}du. \qquad (12)$$

Differentiation with respect to t yields

$$\dot{S} = -rQ + (i + \tau_K)pK + (i + \tau_S)S, \qquad (13a)$$

which has the solution

$$S(t) = \int_t^\infty \left(r(u)Q(K) - (i + \tau_K)pK\right)e^{-(i+\tau_S)(u-t)}du. \qquad (13b)$$

Pre-development land value, V(t), equals

$$V(t) = \max_{K,T} \left\{S(T)e^{-i(T-t)}\right\} - \tau_V \int_t^T V(u)e^{-i(u-t)}du. \qquad (14)$$

Differentiation with respect to t yields

$$\dot{V} = (i + \tau_V)V, \qquad (15a)$$

which has the solution (using (13b) and V(T)=S(T) from (14))

$$V(t) = \max_{K,T} S(T)e^{-(i+\tau_V)(T-t)}$$

$$= \max_{K,T} \left\{\left[\int_T^\infty \left(r(u)Q(K) - (i + \tau_K)pK\right)e^{-(i+\tau_S)(u-T)}du\right]e^{-(i+\tau_V)(T-t)}\right\} \qquad (15b)$$

The developer chooses T and K so as to maximize the expression in curly brackets in (15b). The first-order conditions are

$$\mathrm{T}: \quad \left[-r(T)Q(K)+(i+\tau_K)pK+(\tau_S-\tau_V)V(T)\right]e^{-(i+\tau_V)(T-t)}=0 \tag{16}$$

$$\mathrm{K}: \quad \left[\int_T^{\infty}\left(r(u)Q'(K)-(i+\tau_K)p\right)e^{-(i+\tau_S)(u-T)}du\right]e^{-(i+\tau_V)(T-t)}=0\,. \tag{17}$$

Eq. (16) states that optimal development time occurs when the marginal benefit from postponing development one period equals the marginal cost. The marginal benefit equals the savings from postponing construction cost one period, which equals construction costs times the user cost of capital, $i+\tau_K$, plus the savings in site value tax payments, $(\tau_S-\tau_V)V(T)$. The marginal cost equals the rent forgone. Eq. (17) states that capital should be added to the site up to the point where the discounted value of the rent attributable to the last unit of capital equals the discounted value of the user cost of capital. The post-development residual site value tax has the effect of increasing the post-development discount rate from i to $i+\tau_S$.

The central question to be addressed is whether it is possible to find a neutral property tax system — a set of tax rates ($\tau_V,\ \tau_S,\ \tau_K$) that results in the same T and K as in the absence of taxation, and expropriates a specified proportion of value.

Comparing (2) and (16) gives the following condition for neutrality with respect to the development timing condition:

$$\tau_K pK+(\tau_S-\tau_V)V(T)=0\,. \tag{18a}$$

Using (15b), this becomes

$$\tau_K pK+(\tau_S-\tau_V)\int_T^{\infty}\left(r(u)Q(K)-(i+\tau_K)pK\right)e^{-(i+\tau_S)(u-T)}du=0. \tag{18b}$$

And comparing (3) and (17) gives the following condition for neutrality with respect to the development density decision:[9]

$$\frac{\int_T^\infty r(u)e^{-i(u-T)}du}{\int_T^\infty r(u)e^{-(i+\tau_S)(u-T)}du} = \frac{i+\tau_S}{i+\tau_K}. \tag{19}$$

<u>Proposition 1</u>: For any functions r(t) and Q(K) and exogenous parameters i and p, there is a property tax system that not only achieves neutrality but also expropriates any specified fraction of land value between 0 and 1.

<u>Proof</u>: Hold K and T at their values at the no-tax optimum. Eq. (19) can be rewritten as $\tau_K = \tau_K(\tau_S;T)$. Substitution of this function into (18b) yields the function $\tau_V = \tau_V(\tau_S;T,K)$. Thus, for any τ_S, a unique τ_K and τ_V can be determined that result in (18b) and (19) being simultaneously satisfied. In other words, for any value of τ_S, there is a unique property tax system that is neutral.

From (13b), setting $\tau_S = \infty$ expropriates all of the no-tax site value, while setting $\tau_S = 0$ expropriates none of the no-tax site value since with $\tau_S = 0$, τ_K and τ_V are also zero. From (19), $\tau_K(\tau_S;T)$ is a continuous function of τ_S. From (13b), S(T) is therefore a continuous function of τ_S. Thus, there is a τ_S such that any specified proportion of the no-tax site value between 0 and 1 is expropriated.[10] ■

The general relationship between τ_V, τ_S, and τ_K in a neutral property tax system is complex and is investigated in Appendix 2. A complete characterization of the relationship between

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[9] It is assumed that with the property tax system in place, there is a unique extremum which is interior and is a maximum. The global maximum is then uniquely characterized by the first-order condition.

[10] Observe that the proof generalizes to the situation where i, p, and Q(K) are all functions of time.

τ_V, τ_S, and τ_K for the central case where structure rents grow exponentially over time can, however, easily be obtained. The growth rate of structure rents is denoted by $\eta(>0)$.[11]

Proposition 2: When structure rents grow at a constant rate η, a neutral property tax system has the properties that:

i) $\tau_K = \tau_S\left(-\frac{\eta}{i+\tau_S-\eta}\right)$

ii) $\tau_V = 0$

Proof: i) follows directly from (19). ii) then follows from (18b), after substitution of i). ■

We provide two different intuitive explanations for the results in Proposition 2. The first is casual, the second exact. The residual site value tax system considered in the previous section is a special case of the class of property tax systems considered here, with $\tau = \tau_V = \tau_S$ and $\tau_K = 0$. Recall (proof of Result 5) that that tax system had no effect on the development timing condition but caused the development density condition to change in such a way that (with $\tau > 0$) discourages density. Take the residual site value tax system as the starting point and consider how τ_S, τ_V, and τ_K should be modified to restore neutrality. First, capital should be subsidized to offset the depressing effect of residual site value taxation on development density. But from (16), the subsidization of capital reduces the marginal benefit from postponing development. The development timing condition, which was undistorted with residual site value taxation, becomes distorted, leading to excessively early development. This can be corrected by setting the pre-development land value tax rate below the post-development residual site value tax rate. This intuition suggests that a neutral property tax system which raises positive revenue has $\tau_S > \tau_V$ and $\tau_K < 0$. This intuition is consistent with Proposition 2 which concerns a special case, but is not correct in general (see Appendix 2).

[11] A necessary condition for a local maximum is that rents be growing at development time.

The precise intuition is based on a result that is sufficiently important to present it as:

Proposition 3: When structure rents grow at a constant rate η, the neutral property tax system described in Proposition 2 is equivalent to a site rent tax system with the time-invariant tax rate

$\tau_n = \dfrac{\tau_S}{i + \tau_S - \eta}$.

Proof: Since both tax systems are neutral and hence have the same development time and density, it suffices to demonstrate that the time paths of tax revenue collected under the two tax systems coincide. For both, the tax revenue collected prior to development is zero. After development, the time path of revenue collected under the site rent tax is

$$R(t) = \tau_n \left(r(t)Q(K) - ipK\right). \tag{20}$$

With a property tax system, the time path of revenue collected after development is

$$R(t) = \tau_S S(t) + \tau_K pK$$

$$= \tau_S \frac{r(t)Q(K)}{i + \tau_S - \eta} - \frac{\tau_S \left(i + \tau_K\right)pK}{i + \tau_S} + \tau_K pK \quad \text{(using (13b))}. \tag{21}$$

Now substitute property i) of the neutral property tax system into (21):

$$R(t) = \frac{\tau_S}{i + \tau_S - \eta} \left(r(t)Q(K) - ipK\right). \tag{22}$$

With $\tau_n = \dfrac{\tau_S}{i + \tau_S - \eta}$, the two tax revenue streams are identical. ■

Proposition 3 has an immediate:

Corollary: Under a neutral property tax system, with structure rents growing at a constant rate η: i) at every point in time the ratio of property tax revenue collected to site rent equals $\tau_S / (i + \tau_S - \eta)$; and

ii) the ratio of the present value of property tax revenue collected to the no-tax pre-development land value (for $t < T$) or the no-tax post-development residual site value (for $t > T$) — which is one measure of the proportion of value expropriated by the tax system — equals $\tau_S/(i + \tau_S - \eta)$.

Part i) of the Corollary follows immediately from the proof to Proposition 3. Part ii) follows from part i). The Corollary is related to the second part of Proposition 1. There it was shown that there exists a neutral property tax system that expropriates any desired proportion of the no-tax residual site value. Proposition 3 gives the exact relation between τ_S and the proportion of no-tax site value expropriated, for the special case of a constant growth rate of structure rents.

Another useful result for the situation where structure rents grow at a constant rate is given in:[12]

Proposition 4: Under the neutral property tax system described in Proposition 2, property value at development time, site value at development time, and structure value are in the proportions: $\tau_S - \tau_K$, $-\tau_K$, τ_S.

The derivation of neutral property tax systems presented in this subsection made a large number of simplifying assumptions. Future research should investigate how the results need to be modified when account is taken of time variation in the interest rate, the price of structure capital, and tax rates, as ell as technological change in construction, maintenance and depreciation, the possibility of redevelopment, and uncertainty[13].

II. 2 Discussion

A companion paper (Arnott (2000a)) focuses on practical policy issues related to the results of the previous subsection.[14] This subsection provides a condensed discussion of some of these issues.

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12 This result follows from (18a) and $\tau_V = 0$.

13 Capozza and Li (1994), treating land development as a real option, incorporate uncertainty into the Arnott-Lewis model.

14 One such issue is the treatment of site preparation, servicing, and infrastructure costs. The model assumes that vacant land is instantaneously transformed into a developed property. In fact, however, there are several stages in the development

Since site rent taxation at a time-invariant rate is neutral, why not employ such a site rent tax rather than a more complex neutral property tax system? The primary reason is presumably that site rents are typically unobservable and would be difficult to estimate. While the estimation of pre-development land value, post-development property value, and structure value is by no means trivial, there is a wealth of practical experience to draw on. This observation points to the importance of considering the informational feasibility of implementing alternative property tax systems.

Is implementation of the neutral property tax system derived in the previous subsection informationally feasible? At first glance, the answer would appear negative since the optimal tax rates on a parcel depend on that parcel's future time path of structure rents, which is in practice unknown. While the market does not directly signal expectations concerning future structure rents, it does provide some information relevant to computing the optimal tax rates: prior to development, the market value of a parcel, and immediately after development, development time, development density, and property value. The question is whether this information is sufficient to calculate the set of tax rates that achieves neutrality and raises the desired revenue or expropriates the desired proportion of value. In terms of the model (which entails the assumptions, *inter alia*, that the rent on vacant land is zero and that the interest rate is time-invariant), this information is sufficient if post-development structure rents grow at a constant rate, but not generally otherwise. Let ε denote the desired proportion of value to expropriate, which is defined as the ratio of the value of tax revenue collected to the no-tax land value, both evaluated at development time. With exponential structure rental growth, Proposition 2 implies that $\tau_V = 0$. Proposition 4 and (A2.9´´) with $\tau_V = 0$ that

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process -- site acquisition, preparation, servicing, etc. -- and the overall process takes time. How should tax policy treat these various stages to achieve neutrality?

Another issue concerns zoning. The model assumes that the developer has complete discretion with respect to development timing and density. Zoning, however, may impose inefficient constraints on development timing and density,

$$\tau_S = \frac{i\varepsilon pK}{(1-\varepsilon)pK + S(T)}, \tag{22´}$$

and Proposition 4 that $\tau_K = -\tau_S S(T)/pK$. These results were derived on the assumption that developers take tax rates as parametric. If, however, the government were to compute τ_S and τ_K for a particular property on the basis of *that property's* pK and $S(T)$, the developer would take into account that by altering the timing and density of development he could alter the τ_S and τ_K which would be applied to his property which would undermine neutrality.[15] This problem is easily overcome by basing a particular property's tax rates on the $S(T)/pK$ ratio for "comparable", recently-developed properties.

When however structure rents do not grow at a constant rate, the situation is more complicated. Neutrality in general requires a non-zero tax rate on (pre-development) land value. But prior to development, the only information the market provides on a particular undeveloped property concerning its post-development structure rental stream is its land value. This and the time at which the land value tax is first imposed do not provide enough information to compute τ_V. Intuition suggests that the market is sufficiently uncertain concerning the time path of future structure rents that it has only weak beliefs concerning how the time path of future structure rents will differ from a

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so that second-best efficiency may entail a non-neutral tax system. Relatedly, the ostensible purpose of most zoning restrictions is to internalize externalities. How should these be taken into account?

[15] From (13b),

$$S(T) = \frac{r(T)Q(K)}{i+\tau_S-\eta} - \frac{i+\tau_K}{i+\tau_S}\,pK. \tag{i}$$

Since $\tau_V = 0$, the developer would choose T and K to maximize $S(T)e^{-i(T-t)}$, taking into account the dependence of τ_K and τ_S on T and K. From (22´) and Prop. 4:

$$\frac{i+\tau_K}{i+\tau_S} = 1-\varepsilon. \tag{ii}$$

Substituting (ii) and (22´) into (i) yields

$$S(T) = \frac{r(T)Q(K)}{i-\eta} - \frac{pK\big(1-(1-\varepsilon)\eta\big)}{i-\eta} \tag{iii}$$

constant-growth-rate time path. This suggests that employing a property tax system that would be neutral if the future growth rate of structure rents were constant would normally come close to achieving neutrality. Examination of this conjecture will require extending the model to allow for uncertainty.

The above argument suggests that design of a near-neutral property tax system is informationally feasible. What of administrative feasibility? The analysis of the previous subsection applied to an isolated property. If the tax system were to be applied as modeled, every property would have its own time-invariant tax rate on post-development residual site value and subsidy rate on structure value, which would be very cumbersome. And since all tax rates on developed properties would have been set in the past and since all tax rates on vacant land would be zero, the government would have no discretion to raise or lower tax revenues in the short run. Clearly, administrative feasibility requires adapting the property tax system analyzed in the previous subsection.

The objective therefore is to find a property tax system that is informationally and administratively feasible and that comes close to being neutral. Consider the following simple adaptation of the neutral property tax system analyzed earlier. After development, impose a tax on residual site value along with a structure investment tax credit. The tax rate on pre-development land value would be zero; the tax rate on post-development residual site value for a particular class of properties would be set annually according to the government's revenue requirements, etc.; and for that class of properties the tax credit rate on structure investment would be set annually with the objective of coming as close as possible to achieving neutrality with respect to the timing and density of development. Implementation of such a tax system would require addressing a host of practical issues: How finely should the tax credit rate on structure investment be varied over space, if at all? How

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Thus, basing a particular property's τ_S and τ_K on that property's pK and $S(T)$ would increase the effective price of capital, should the transition from the current system to this system be designed so as to achieve a smooth revenue stream, to avoid causing a building boom or bust, and to be politically acceptable, which requires among other things not generating substantial capital losses on any major class of properties? And how should the tax credit rate on structure investment be determined? This tax system is only one of many that might attain the best balance between practicability and deviation from neutrality, and has been presented more as a basis for discussion than as an advocated proposal.

III. Conclusion

The paper started by providing a synthetic overview of the literature on site/land taxation. The orthodox view is that the land value taxation is non-distortionary. The basic idea is that the value of land is independent of current decisions concerning its use. If that is the case, taxation of such value is then regarded by the developer as a lump-sum tax, and does not therefore affect his decisions concerning its use. No contributor to the modern, mainstream literature on the subject disputes this. The disagreement instead centers on how land should be valued for property tax purposes *after* it has been developed — when there is a durable and immobile structure on the site. Since there is no market for such land, its value is not logically determinable. There are two broad points of view concerning how the value of developed land should be imputed for property tax purposes. Defenders of the orthodoxy argue that site value should be defined in such a way that its taxation is neutral. There are many ways this can be done. One such definition was treated, *raw site value* — what the site would sell for *were it undeveloped*, even after it has in fact been developed. The problem with using this definition is that post-development raw site value would be sufficiently difficult to estimate that assessment would likely be inequitable, capricious, and subject to abuse. The revisionists have

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which is inconsistent with neutrality.

employed an alternative definition of land value for a built-on site: property value minus structure value, which is termed *residual site value*. Property value can be estimated using current assessment practice based on hedonic analysis, while structure value can be estimated by applying an estimated depreciation rate to original construction costs. Residual site value could therefore be estimated with relative ease (thought perhaps also with considerable imprecision). However, using residual site value as a basis for taxation violates neutrality; in particular, holding fixed development time, it discourages density.

Reasonable men may differ concerning which of the two broad approaches to site value taxation is the more promising. Vickrey, whose logic is always impeccable, favored a definition which comes as close as is administratively feasible to preserving neutrality, and his position is shared by defenders of the orthodoxy. Another group of economists who are less sanguine concerning the feasibility of developing acceptable procedures for estimating raw site value come down on the side of taxing residual site value.

This paper contributed to the literature on residual site value taxation. Contributors to this literature have demonstrated that residual site value tax is distortionary, but have not taken the next step of asking the question: Is it possible to design a *property* tax system employing the residual definition of site value for built-on land that is neutral? That was the central question addressed in this paper.

A property tax system was defined as a triple of linear, time-invariant taxes: a tax on pre-development land value at rate τ_V, a tax on post-development residual site value at rate τ_S, and a tax on post-development structure value at rate τ_K. To address the question, a partial equilibrium model was employed which looked at a single developable site. Among the simplifying assumptions made were that once a site is developed at a particular density it remains at that density forever, and that the rent on undeveloped land is zero. The main result was that for this model there is indeed a combination of the three tax rates that raises a given level of discounted tax revenues and achieves neutrality. The basic intuition is simple. The government has three objectives — not distorting the development timing decision, not distorting the development density condition, and extracting a pre-determined proportion of value — and three instruments to achieve these objectives. This intuition suggests that the neutrality result extends to considerably more realistic models than the one employed in this paper.

The paper then calculated the three tax rates that achieve neutrality for the central case in which the structure rental growth rate is constant over time. The tax rate on pre-development land value should be zero, the tax rate on post-development residual site value should be set so as to achieve the desired expropriation of value, and the structure value tax rate should be negative. One intuition is that, under the assumptions made, this property tax system is equivalent to a tax on net site rent at a time-invariant rate, which was earlier shown to be neutral.

The paper then briefly discussed how the insights from the theoretical analysis might be applied to the design of practical property tax systems, taking into account considerations of informational and administrative feasibility, and of political acceptability. Two general points were made. The first was that the model requires considerable elaboration before it can be confidently employed as a basis for policy and that even then many issues outside the model would need to be considered. The second was that, these cautions notwithstanding, it should be possible to design a practicable property tax system that is substantially less distortionary[16] than the current property tax system in place throughout most of North America, which effectively taxes pre-development land value and post-development site and

[16] The existing literature examines the deadweight loss of property taxation under the assumption that rents rather than values are taxed. Another companion paper (Arnott, 2000b) investigates the deadweight loss of alternative property tax systems taking into account that it is values not rents that are taxed.

structure values at the same rate.[17] A sample system was put forward as a basis for discussion: Tax exemption for land prior to development, and after development taxation of residual site value combined with a structure investment tax credit.

The literature on property taxation, to which the paper has contributed, has evolved largely independently of other important developments in public economics. There is an extensive literature on neutral capital taxation (e.g., Samuelson (1964), King and Fullerton (1984), and Boadway, Bruce, and Mintz (1984)). The two literatures should be integrated, not only to develop results on neutral capital *c* property taxation, but also to investigate second-best efficient property taxation when capital taxation is distorted, and *vice versa*. There is also an extensive literature on the design of optimal tax systems, which takes into account the equity-efficiency tradeoffs produced by asymmetries in information. It is time for the property tax to be considered as one component of a broad tax system rather than being examined in isolation. Since property is an asset, the portfolio effects of property taxation need to be considered as well.

[17] This statement needs to be qualified. Agricultural land is often taxed on the basis of its "agricultural land value" -- what the value of the land would be worth if it were held in agricultural use forever. In the limit as agricultural land rent approaches zero, this corresponds to a zero tax rate on the market value of pre-development land.

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Appendix 1

Proof of Result 3:

Prior to development, raw site value equals the value of the vacant land, $V(t)$. After development, raw site value equals the value of the land were it still undeveloped.

Using (6), the value of vacant land for t<T is

$$V(t) = \max_{K,T} \left[\int_T^\infty r(u)Q(K)e^{-i(u-t)}du - pKe^{-i(T-t)} - \tau_S \int_t^\infty S(u)e^{-i(u-t)}du \right]$$

$$= \max_{K,T} \left[\int_T^\infty r(u)Q(K)e^{-i(u-t)}du - pKe^{-i(\tau-t)} - \tau_S \int_t^T V(u)e^{-i(u-t)}du - \tau_S \int_T^\infty \Phi(u)e^{-i(u-t)}du \right]. \quad \text{(A1.1a)}$$

The value of the land at $t \geq T$ if, hypothetically, it were still undeveloped is

$$\begin{aligned} \Phi(t) &= \int_{\hat{T}(t)}^\infty r(u)Q\left(\hat{K}(t)\right)e^{-i(u-t)}du - p\hat{K}(t)e^{-i\left(\hat{T}(t)-t\right)} - \tau_S \int_t^\infty \Phi(u)e^{-i(u-t)}du \\ &= \int_{\hat{T}(t)}^\infty \left(r(u)Q\left(\hat{K}(t)\right) - ip\hat{K}(t)\right)e^{-(i+\tau_S)(u-t)}du, \end{aligned} \quad \text{(A1.1b)}$$

where $\hat{T}(t)$ is the profit-maximizing time to develop the land *conditional on its being undeveloped at time t*, and $\hat{K}(t)$ is defined analogously. $\hat{K}(t)$ is solved for from the first-order condition for density:

$$\int_{\hat{T}(t)}^\infty \left(r(u)Q'\left(\hat{K}(t)\right) - ip\right)e^{-(i+\tau_S)(u-t)}du = 0. \quad \text{(A1.1c)}$$

In a continually growing economy $(\acute{Y} > 0)$, if land were still undeveloped at t>T, it would be profit-maximizing to develop it right away, in which case (A1.1b) simplifies to

$$\Phi(t) = \int_t^\infty \left(r(u)Q\left(\hat{K}(t)\right) - ip\hat{K}(t)\right)e^{-(i+\tau_S)(u-t)}du, \quad \text{(A1.1b')}$$

$\hat{K}(t)$ being given implicitly by (A1.1c) with $\hat{T}(t)$=t. If the economy does not grow continually, the calculation of $\hat{T}(t)$ in general requires non-local analysis.

Since the developer in fact develops at t=T, post-development raw site value is independent of his actions. Accordingly, he views post-development raw site value tax payments as lump-sum taxes.

Pre-development raw site value, meanwhile, is the market value and does depend on the profit-maximizing T and K.

Define Z(T) to be the sum of post-development raw site value tax payments, discounted to T:

$$Z(T) = \tau_S \int_T^{\infty} \Phi(u) e^{-i(u-T)} du . \qquad \text{(A1.2a)}$$

Substituting (A1.2a) into (A1.1a) yields

$$V(t) = \max_{K,T} \left[\int_T^{\infty} r(u) Q(K) e^{-i(u-t)} du - pKe^{-i(T-t)} - \tau_S \int_t^T V(u) e^{-i(u-t)} du - Z(T) e^{-i(T-t)} \right]. \quad \text{(A1.1a}'\text{)}$$

Differentiation with respect to t gives

$$\dot{V} = (i + \tau_S) V$$

and

$$V(t) = \max_{K,T} \left[\int_T^{\infty} r(u) Q(K) e^{-i(u-T)} du - pK - Z(T) \right] e^{-(i+\tau_S)(T-t)} . \qquad \text{(A1.1a}''\text{)}$$

It is easy to see from $(\text{A1.1a}'')$ that the first-order condition for density is independent of τ_S. The first-order condition for development timing is

$$-(i + \tau_S) V(T) - r(T) Q(K) + i \int_T^{\infty} r(u) Q(K) e^{-i(u-T)} du - Z'(T) = 0 . \qquad \text{(A1.3a)}$$

Substituting $(\text{A1.1a}'')$ evaluated at T into (A1.3a) yields

$$-\tau_S V(T) - r(T) Q(K) + ipK + iZ(T) - Z'(T) = 0 . \qquad \text{(A1.3b),}$$

Finally, substituting $Z'(T) = -\tau_S \Phi(T) + iZ(T)$ (from (A1.2a)) and $V(T) = \Phi(T)$, (A1.3b) reduces to the first-order condition without the raw site value tax. Thus, the raw site value tax is neutral. ■

To understand this result, consider first the development density condition. Turn to $(\text{A1.1a}')$. For the development density condition to be unaffected by the site value tax, the derivative of the last two terms with respect to K must equal zero. The derivative of the second last term equals zero since

V(u) is the maximized value of discounted net revenue with respect to K, implying $\frac{\partial V(u)}{\partial K} = 0$. And the derivative of the last term equals zero since, with development time fixed, post-development raw site value tax payments are independent of development density. Consider next the development timing condition. For this condition too to be unaffected by the site value tax, the derivative of the last two terms of (A1.1a′) with respect to T must equal zero. Using (A1.2a), this derivative is

$$-\tau_S \int_t^T \frac{\partial V(u)}{\partial T} e^{-i(u-t)} du - \tau_S \left(V(T) - \Phi(T)\right) e^{-i(T-t)}.$$

The first term equals zero since $V(u)$ is the maximized value of discounted net revenue with respect to T, implying $\frac{\partial V(u)}{\partial T} = 0$, and the second equals zero since $V(T) = \Phi(T)$.

Appendix 2

General Relationship between τ_V, τ_S, and τ_K in a neutral property tax system

The two neutrality conditions are (18b) and (19). First, the revenue condition will be derived. Then the properties of the three-equation system shall be investigated.

a) <u>the revenue condition</u>

To simplify the notation somewhat, let t=0 denote the time at which the land value tax is first applied with T>0.

Let $R(T)$ denote the value of revenue collected, evaluated at time T:

$$R(T) = \tau_V \int_0^T V(t)e^{i(T-t)}dt + \tau_K \int_T^\infty pKe^{-i(t-T)}dt + \tau_S \int_T^\infty S(t)e^{-i(t-T)}dt. \qquad \text{(A2.1)}$$

From (15a)

$$\begin{aligned} \tau_V \int_0^T V(t)e^{i(T-t)}dt &= \int_0^T \tau_V V(T)e^{-(i+\tau_V)(T-t)}e^{i(T-t)}dt \\ &= V(T)\left(1-e^{-\tau_V T}\right) \end{aligned} \qquad \text{(A2.2)}$$

From (12)

$$\tau_S \int_T^\infty S(t)e^{-i(t-T)}dt = -S(T) + \int_T^\infty \left(rQ - (i+\tau_K)pK\right)e^{-i(t-T)}dt. \qquad \text{(A2.3)}$$

And

$$\int_T^\infty pKe^{-i(t-T)}dt = \frac{pK}{i}. \qquad \text{(A2.4)}$$

Combining (A2.1) - (A2.4) yields

$$R(T) = V(T)\left(1-e^{-\tau_V T}\right) + \frac{\tau_K pK}{i} - S(T) + \int_T^\infty \left(rQ - (i+\tau_K)pK\right)e^{-i(t-T)}dt$$

$$= -V(T)e^{-\tau_V T} + \int_T^\infty rQe^{-i(t-T)}dt - pK \qquad \text{(using } S(T) = V(T)\text{)}$$

$$= -\left(\int_T^\infty \left(rQ - (i+\tau_K)pK\right)e^{-(i+\tau_S)(t-T)}dt\right)e^{-\tau_V T} + \int_T^\infty rQe^{-i(t-T)}dt - pK. \text{ (using (15b))} \quad \text{(A2.5)}$$

b) <u>the three equations</u>

Define

$$A \equiv \int_T^\infty r(t)e^{-i(t-T)}dt \qquad B(\tau_S) \equiv \int_T^\infty r(t)e^{-(i+\tau_S)(t-T)}dt. \quad \text{(A2.6)}$$

Rewrite the three equations using (A2.6). Eq. (19) becomes

$$A(i+\tau_K) - (i+\tau_S)B(\tau_S) = 0. \quad \text{(A2.7)}$$

Equation (18b) becomes

$$\tau_K pK + (\tau_S - \tau_V)\left(B(\tau_S)Q - \frac{i+\tau_K}{i+\tau_S}pK\right) = 0. \quad \text{(A2.8)}$$

And eq. (A2.5) becomes

$$R(T) = -\left(B(\tau_S)Q - \frac{i+\tau_K}{i+\tau_S}pK\right)e^{-\tau_V T} + AQ - pK. \quad \text{(A2.9)}$$

Now substitute (A2.7) into (A2.8) and (A2.9). Then the equations system can be written as

$$A(i+\tau_K) - (i+\tau_S)B(\tau_S) = 0 \quad \text{(A2.7)}$$

$$\tau_K pK + (\tau_S - \tau_V)\left(\frac{i+\tau_K}{i+\tau_S}\right)(AQ - pK) = 0 \quad \text{(A2.8´)}$$

$$R(T) = (AQ - pK)\left(1 - \frac{i+\tau_K}{i+\tau_S}e^{-\tau_V T}\right). \quad \text{(A2.9´)}$$

Note that AQ-pK is site value at development time in the pre-tax situation. Thus,

$$\varepsilon \equiv \frac{R(T)}{AQ - pK} \quad \text{(A2.10)}$$

is the ratio of the value of tax collected evaluated at development time to the pre-tax site value at development time, which is the measure employed of the proportion of site value expropriated through the tax system. Define $\tilde{\eta}$ implicitly as

$$i - \tilde{\eta} = r(T)/A. \qquad \text{(A2.11)}$$

Then the development timing condition in the pre-tax situation, (2), can be written as

$A(i - \tilde{\eta})Q - ipK = 0$, so that

$$AQ - pK = pK\left(\frac{i}{i - \tilde{\eta}} - 1\right) = \frac{pK\tilde{\eta}}{i - \tilde{\eta}}. \qquad \text{(A2.12)}$$

Using (A2.10) and (A2.12), (A2.7), (A2.8´), and (A2.9´) can be rewritten as

$$A(i + \tau_K) - (i + \tau_S)B(\tau_S) = 0 \qquad \text{(A2.7)}$$

$$\tau_K + (\tau_S - \tau_V)\left(\frac{i + \tau_K}{i + \tau_S}\right)\frac{\tilde{\eta}}{i - \tilde{\eta}} = 0 \qquad \text{(A2.8}''\text{)}$$

$$1 - \left(\frac{i + \tau_K}{i + \tau_S}\right)e^{-\tau_V T} - \varepsilon = 0. \qquad \text{(A2.9}''\text{)}$$

This set of three equations characterizes the set of (τ_V, τ_S, and τ_K) that achieve neutrality and expropriate a proportion ε of site value.

Before proceeding, two intermediate results are established: $i + \tau_K > 0$ and $i + \tau_S > 0$. From (17), if $i + \tau_K < 0$, the profit-maximizing amount of capital to employ would be infinite, which is inconsistent with neutrality; hence, $i + \tau_K > 0$. Since $A > 0$, $B(\tau_S) > 0$ and $i + \tau_K > 0$, it follows from (A2.7) that $i + \tau_S > 0$.

c) $\underline{\tau_K = \tau_K(\tau_S)}$

Observe that (A2.7) gives τ_K as a function of τ_S: $\tau_K = \tau_K(\tau_S)$ when τ_S=0, A=B(0) so that τ_K=0; thus, $\tau_K(\tau_S)$ passes through the origin in τ_S-τ_K space. Also,

$$\left.\frac{d\tau_K}{d\tau_S}\right|_{(A2.7)} = \frac{B(\tau_S) + (i+\tau_S)B'(\tau_S)}{A}. \tag{A2.13}$$

From (A2.6), defining u=t-T, $B(\tau_S) = \int_0^\infty r(u)e^{-(i+\tau_S)u}du$, so that

$$B'(\tau_S) = -\int_0^\infty r(u)ue^{-(i+\tau_S)u}du < 0$$

$$= \left(\frac{r(u)ue^{-(i+\tau_S)u}}{i+\tau_S}\right)_0^\infty - \int_0^\infty \frac{(\dot{r}u + r)}{(i+\tau_S)}e^{-(i+\tau_S)u}du \text{ (integration by parts).} \tag{A2.14}$$

Substituting (A2.14) into (A2.13) yields

$$\left.\frac{d\tau_K}{d\tau_S}\right|_{(A2.7)} = -\frac{1}{A}\int_0^\infty \dot{r}ue^{-(i+\tau_S)u}du. \tag{A2.15}$$

This is ambiguous in sign. In a growing economy, however, one expects $\dot{r} > 0$, except for downturns in the business cycle. Thus, "normally" $\left.\frac{d\tau_K}{d\tau_S}\right|_{(A2.7)} < 0$.

(AS-1): $\int_0^\infty \dot{r}ue^{-(i+\tau_S)u}du > 0 \text{ for } u \equiv t - T$.

<u>Proposition A1</u>: Under (AS-1), $\left.\frac{d\tau_K}{d\tau_S}\right|_{(A2.7)} < 0$.

Differentiating (A2.15) with respect to τ_S gives

$$\left.\frac{d^2\tau_K}{d\tau_S^2}\right|_{(A2.7)} = \frac{1}{A}\int_0^\infty \dot{r}u^2e^{-(i+\tau_S)u}du.$$

(AS-2): $\int_0^\infty \dot{r}u^2e^{-(i+\tau_S)u}du > 0 \text{ for } u \equiv t - T$.

<u>Proposition A2</u>: Under (AS-2), $\left.\frac{d^2\tau_K}{d\tau^2_S}\right|_{(A2.7)} > 0$.

Figure A1 plots the relationship between τ_K and τ_S under (AS-1) and (AS-2).

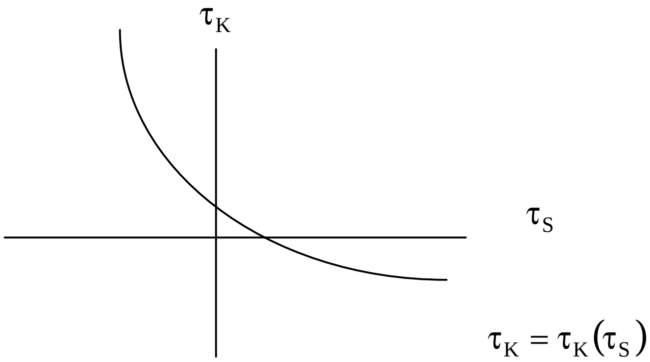


Figure A1

d) $\underline{\tau_V = \bar{\tau}_V(\tau_S)}$

Substituting (A2.7) into (A2.9′′) gives τ_V as a function of $\tau_S\left(\tau_V = \bar{\tau}_V(\tau_S)\right)$:

$$\tau_V = \frac{1}{T}\left(\ln B(\tau_S) - \ln(1-\varepsilon)A\right). \tag{A2.16}$$

Differentiating (A2.16) yields

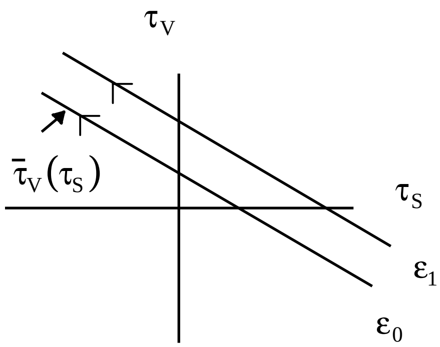
$$\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.16)} = \frac{B'}{BT} < 0 \tag{A2.17}$$

Also,

$$\left.\frac{d^2\tau_V}{d\tau_S{}^2}\right|_{(A2.16)} = \frac{B''}{BT} - \frac{(B')^2}{B^2T},$$

which is in general ambiguous in sign.

Figure A2 plots this function for different values of ε.



$\varepsilon_1 > \varepsilon_0$ Figure A2

The loci are parallel and loci further northeast from the origin correspond to higher levels of expropriation.

e) $\underline{\tau_V = \bar{\bar{\tau}}_V(\tau_S)}$

Substituting (A2.7) into (A2.8″) gives another relationship between τ_V and $\tau_S \left(\tau_V = \bar{\bar{\tau}}_V(\tau_S)\right)$:

$$\left(\frac{(i+\tau_S)B(\tau_S)}{A} - i\right) + (\tau_S - \tau_V)\frac{B(\tau_S)}{A}\frac{\tilde{\eta}}{i-\tilde{\eta}} = 0$$

or

$$\tau_V = \frac{i(i-\tilde{\eta})}{\tilde{\eta}}\left(1 - \frac{A}{B(\tau_S)}\right) + \tau_S \frac{i}{\tilde{\eta}}. \tag{A2.18}$$

Note first that when τ_S=0, A=B, which implies that τ_V=0. Thus, $\bar{\bar{\tau}}_V(\tau_S)$ passes through the origin in $\tau_S - \tau_K$ space. Also,

$$\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.18)} = \frac{i}{\tilde{\eta}}\left((i-\tilde{\eta})\frac{A}{B^2}B' + 1\right). \tag{A2.19}$$

Now define $\hat{\eta}(\tau_S)$ implicitly by

$$B = \int_0^\infty r(u)e^{-(i+\tau_S)u}du = \int_0^\infty r(0)e^{-(i+\tau_S-\hat{\eta}(\tau_S))u}du \tag{A2.20a}$$

and $\hat{\hat{\eta}}(\tau_S)$ implicitly by

$$B' = -\int_0^\infty r(u)ue^{-(i+\tau_S)u}du = -\int_0^\infty r(0)ue^{-\left(i+\tau_S-\hat{\hat{\eta}}(\tau_S)\right)u}du. \tag{A2.20b}$$

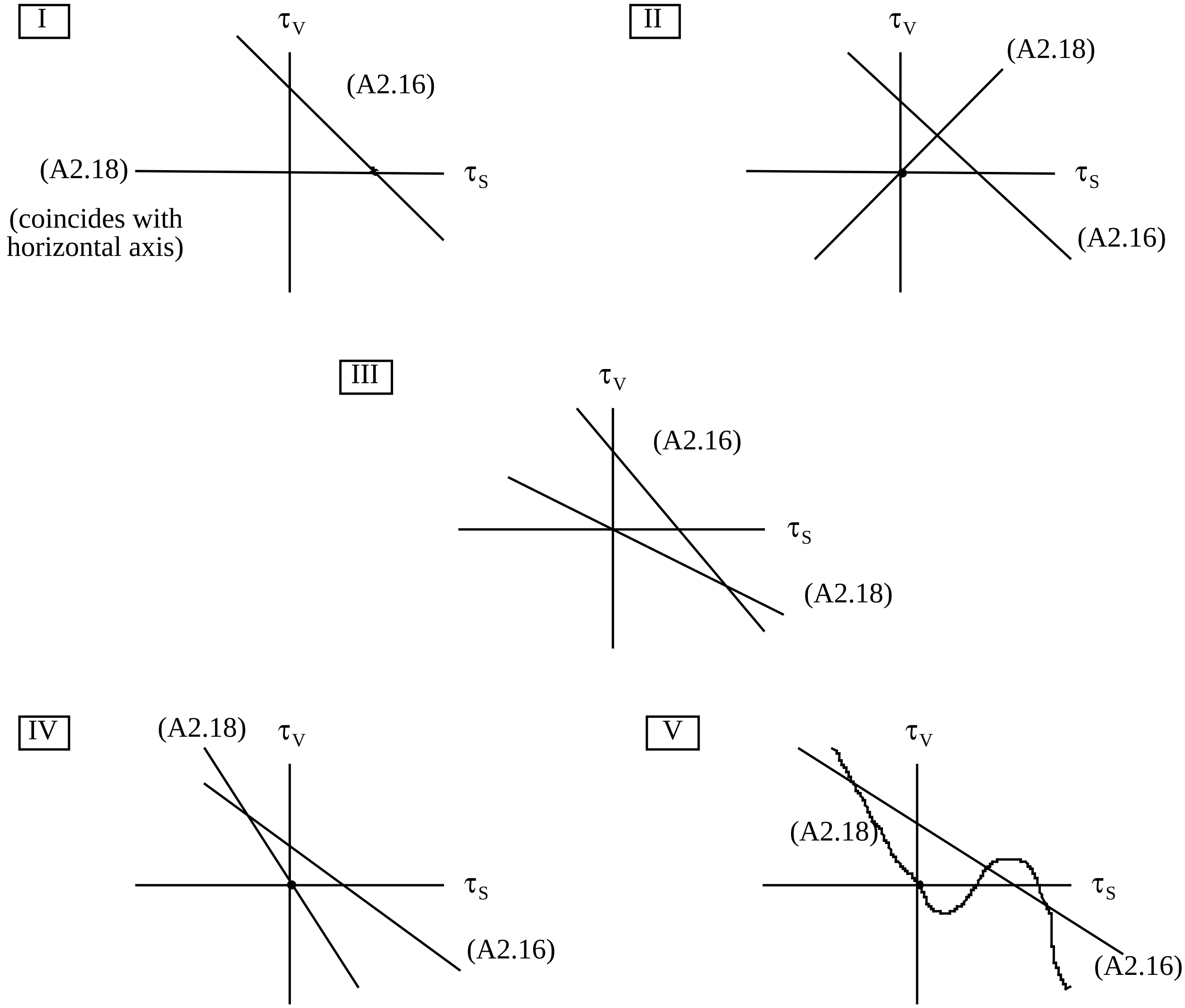
Both $\hat{\eta}(\tau_S)$ and $\hat{\hat{\eta}}(\tau_S)$ are weighted average structure rental growth rates. Because B' contains the extra u inside the integral, the calculation of $\hat{\hat{\eta}}(\tau_S)$ puts more weight on later periods than does $\hat{\eta}(\tau_S)$. Thus $\hat{\eta}(\tau_S) \gtrless \hat{\hat{\eta}}(\tau_S)$ if the structure rental growth rate $\begin{pmatrix}\text{falls}\\ \text{rises}\end{pmatrix}$ over time. Substituting (A2.20a), (A2.20b), and the definitions of $\hat{\eta}$ and $\hat{\hat{\eta}}$ into (A2.19) yields

$$\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.18)} = \frac{i}{\tilde{\eta}}\left(-\left(\frac{i+\tau_S-\hat{\eta}(\tau_S)}{i+\tau_S-\hat{\hat{\eta}}(\tau_S)}\right)^2+1\right). \tag{A2.21}$$

If structure rental growth is exponential, $\hat{\eta}(\tau_S)=\hat{\hat{\eta}}(\tau_S)$ so that $\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.18)} = 0$. If the structure rental growth rate $\begin{pmatrix}\text{falls}\\ \text{rises}\end{pmatrix}$ over time, $\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.18)} \gtrless 0$, and when the structure rental growth rate is non-monotonic over time $\bar{\bar{\tau}}_V(\tau_S)$ need not be monotonic. These results are sufficiently important to record.

<u>Proposition A3</u>: $\left.\frac{d\tau_V}{d\tau_S}\right|_{(A\,2.18)}$ has the same sign as $\hat{\eta}(\tau_S)-\hat{\hat{\eta}}(\tau_S)$. Thus, if $\hat{\eta}(\tau_S) \overset{>}{\underset{<}{=}} \hat{\hat{\eta}}(\tau_S)$ for all τ_S, then $\bar{\bar{\tau}}_V(\tau_S)$ is $\begin{Bmatrix}\text{monotonically increasing}\\ \text{constant}\\ \text{monotonically decreasing}\end{Bmatrix}$.

Figure A3 plots $\bar{\tau}_V(\tau_S)$ ((A2.16)) with $\varepsilon > 0$ and $\bar{\bar{\tau}}_V(\tau_S)$ ((A2.18)) for five cases.



Case I depicts the situation with exponential structure rental growth, which was treated in the main body of the paper. Neutrality entails $\tau_S > 0$, $\tau_V = 0$, and (since (AS-1) is satisfied) $\tau_K < 0$. Case II depicts a maturing city in which the growth rate of structure rents is positive but falling over time. Neutrality entails $\tau_S > 0$, $\tau_V > 0$, and (since (AS-1) is satisfied) $\tau_K < 0$. Cases III and IV depict incipient boom towns in which the structure rental growth rate is increasing over time (and positive since the structure rental growth rate at u=0 is positive from the second-order condition of the developer's profit maximization problem). In Case III, $\tau_S > 0$, $\tau_V < 0$, and (since (AS-1) is satisfied) $\tau_K < 0$; and in case IV, $\tau_S < 0$, $\tau_V > 0$, and (since (AS-1) is satisfied) $\tau_K > 0$. Case V demonstrates the possibility of multiple neutral property tax systems satisfying a particular revenue requirement.

The above line of analysis can be extended straightforwardly to more complex situations, for example where the interest rate varies over time and where there is technical change in construction. The extension to treat uncertainty — for example where the time path of structure rents follows a stochastic process — will be more difficult.